UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: February 2020

Module Number: ME 6302 **Module Name: Automatic Control Engineering**

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1. a) Figure Q1.a and Q1.b show two views of a mechanical reduction gearbox having one pair of gears. In the figures, 'a' and 'b' represents the pitch circle diameters of input and output gears. The reduction ratio is $n = {}^{b}/_{a}$. Damping coefficients of the input shaft and the output shaft are $C_m$and C_o, and the inertias are $I_m$and I_o, respectively. The input shaft receives a torque of $T_m(t)$ from a motor and the corresponding position of the output shaft position is $\theta_o(t)$. The gear tooth reaction force is $X(t)$.

i. Obtain the differential equation of the input shaft, which describes the gear tooth reaction force in terms of input shaft parameters.

[2.0 Marks]

ii. Obtain the differential equation of the output shaft, which describes the gear tooth reaction force in terms of output shaft parameters.

[2.0 Marks]

iii. Obtain the kinematic relationships, which describe position, angular velocity and angular acceleration between input and output shafts. (Hint: Kinematics is the study of motion without considering forces/torques causing that motion. Here you must obtain input and output relationships considering gear ratio but without considering torques/forces, inertias, or damping.)

[1.0 Mark]

iv. Use the above obtained relationships and deferential equations to derive the differential equation of the gearbox in terms of input torque, output position, damping constants, inertias and the gear ratio.

[2.0 Marks]

b) Transfer function of a DC motor is shown below.

$G(s) = \frac{\omega(S)}{V(S)} = \frac{k_m}{\tau_m S+1}$ where $\omega(S), V(S), k_m$ and τ_m denote shaft speed, supply voltage, motor constant and time constant, respectively. When the supply voltage v(t) of the motor is kept at 5 V, the steady-state speed of the motor is 80 rad/s. When the motor is switched ON at t=0 s with a step input of 5 V, the motor takes 3 s to reach 40 rad/s starting from 0 rad/s speed. Find the values of k_m and τ_m.

[5.0 Marks]

Q2. a) The dynamics of an open-loop system is given by the following transfer function. $G(s) = \frac{1}{S(S+8)}$

i. Draw the block diagram of the respective closed-loop system having unity negative feedback and a single proportional controller having gain K.

[1.0 Mark]

ii. Obtain the characteristic equation corresponding to the system obtained in Q2.a(i.)

[1.0 Mark]

iii. Obtain the break-away point of root locus and the corresponding value of the K.

[2.0 Marks]

iv. Find the roots of the characteristic quotation when K has values 0, 8, 24, and 32.

[2.0 Marks]

v. Draw the root locus of the system. (Use your answer book to draw the root locus indicating important points in the s-plane. Graph sheets are not necessary)

[3.0 Marks]

b) Figure Q2 shows the root locus of a SISO LTI system. Obtain the open-loop zeros and open-loop poles of the system, and derive the transfer function. Assuming a proportional-only controller is used to control this plant, what is the possible approximate range of values for the gain to make the plant stable.

[3.0 Marks]

Q3. a) A plant has a transfer function with two poles located at $s = -2$ and $s = -4$. This system has no zeros and the forward gain of the transfer function is 1.

i. Draw the block diagram of the closed-loop negative feedback system with a single feedback gain, and obtain the characteristic equation.

[1.0 Mark]

ii. If 4% overshoot and 2% settling time under 2.5 s are expected from the plant, obtain the required roots of the characteristic equation.

[3.0 Marks]

iii. According to the results obtained in above Q3.a(ii.), show that a proportional-only feedback gain is not sufficient to achieve the expected performance of Q3.a(ii.).

[1.0 Mark]

iv. To obtain the required performance, the system needs additional control system blocks. Draw the complete system with additional blocks included. Clearly name the blocks and mention their respective transfer function inside the blocks using symbolic variables.

[1.0 Mark]

v. If the compensator used to achieve the expected performance as given in

Q3.a(ii), has zero located at $(-6, j0)$, find the location of the pole of the compensator.

[4.0 Marks]

b) State the important characteristics of lead, lag and notch compensators in point form.

[2.0 Marks]

Q4. a) Dynamics of a given plant are modeled using the following transfer function.

$$G(s) = \frac{1}{S(S+2)(S+4)}$$

This plant is controlled suing a proportional only controller C(S) and has a negative feedback gain of 1.

i. Draw the closed-loop system block diagram and obtain the open-loop transfer function of the system.

[1.0 Mark]

ii. Obtain the open-loop frequency response expressions of the system in polar form.

[2.0 Marks]

iii. Find the value of the gain K when one or more system poles are located on the imaginary axis of the s-plane.

[3.0 Marks]

iv. Use the graph paper attached at the end of this question paper to draw the polar plot of the system when ω is varied from zero to infinity. (Use several checkpoints to identify the path of the polar plot).

[2.0 Marks]

v. The information in Q4.a(iv) above can be represented using a Bode plot. What is the main advantage of using the Bode plot over the polar plot?

[1.0 Mark]

b) Explain the procedure to find the gains of a PID controller using the Ziegler Nichols method for a plant with an unknown transfer function, assuming the Ziegler Nichols method is applicable to the plant.

[3.0 Marks]

Q5. Figure Q5 shows a rotary inverted pendulum system and the corresponding free body diagram. The motor of the system rotates the arm and the pendulum is pivoted to the end of the arm as shown. The pivot joint is free to rotate and has negligible friction and damping. The goal of a proposed control system is to keep the pendulum upright by controlling the arm of the system. Without active control, the pendulum will not stay upright. The system uses an armature-controlled brushed permanent magnet DC motor.

a) Derive the differential equations describing electrical and mechanical domains of the motor and convert it to the Laplace domain assuming zero initial conditions. (Note: The DC motor has an inductance L, resistance R, Inertia J, damping constant B, supply voltage v(t), shaft speed w(t), Torque T, back emf constant k_e and torque constant k_t)

[3.0 Marks]

b) Build a non-linear system dynamics model to simulate this rotary inverted pendulum system, using a block diagram. In your answer, include details about critical configurations, locations of the sensors, joints, and forces. State the expected function of each of the blocks used. Assume that the input to the simulator is the torque and the output is the pendulum angle at this stage.

[4.0 Marks]

c) Indicating the system modeled in Q5.b using a single block, draw a suitable closed-loop control system using a block diagram to keep the pendulum at the upright position. Clearly indicate the controller you are using with reasons.

[1.0 Mark]

d) The motor driver of DC motor has a current limit of ±5 A and a dead band of ±0.1 A. The DC motor accepts current as input and outputs torque. Include these details and redraw an improved model.

[2.0 Marks]

e) Suggest a suitable sensor to measure the angle of the pendulum with sufficient accuracy and precision. If you are to implement this system as a physical prototype, what is the expected accuracy?.

[2.0 Marks]

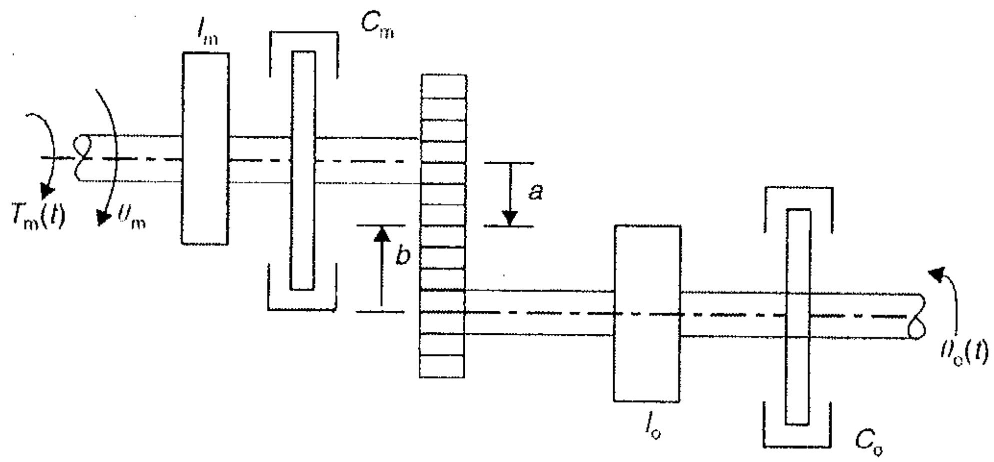


Figure Q1.a

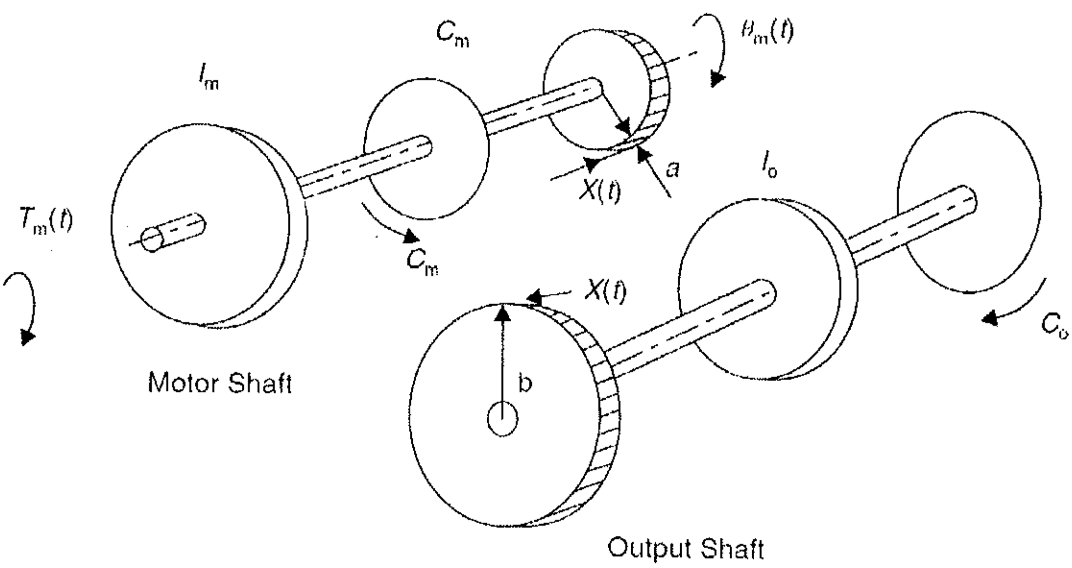


Figure Q1.b

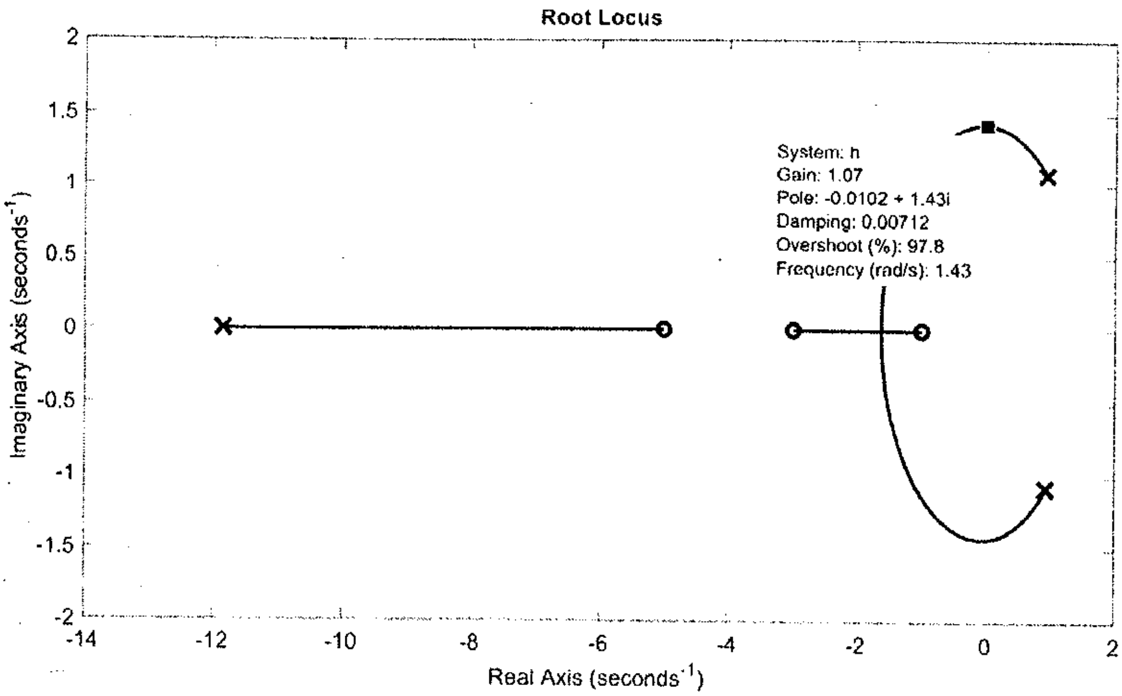


Figure Q2.

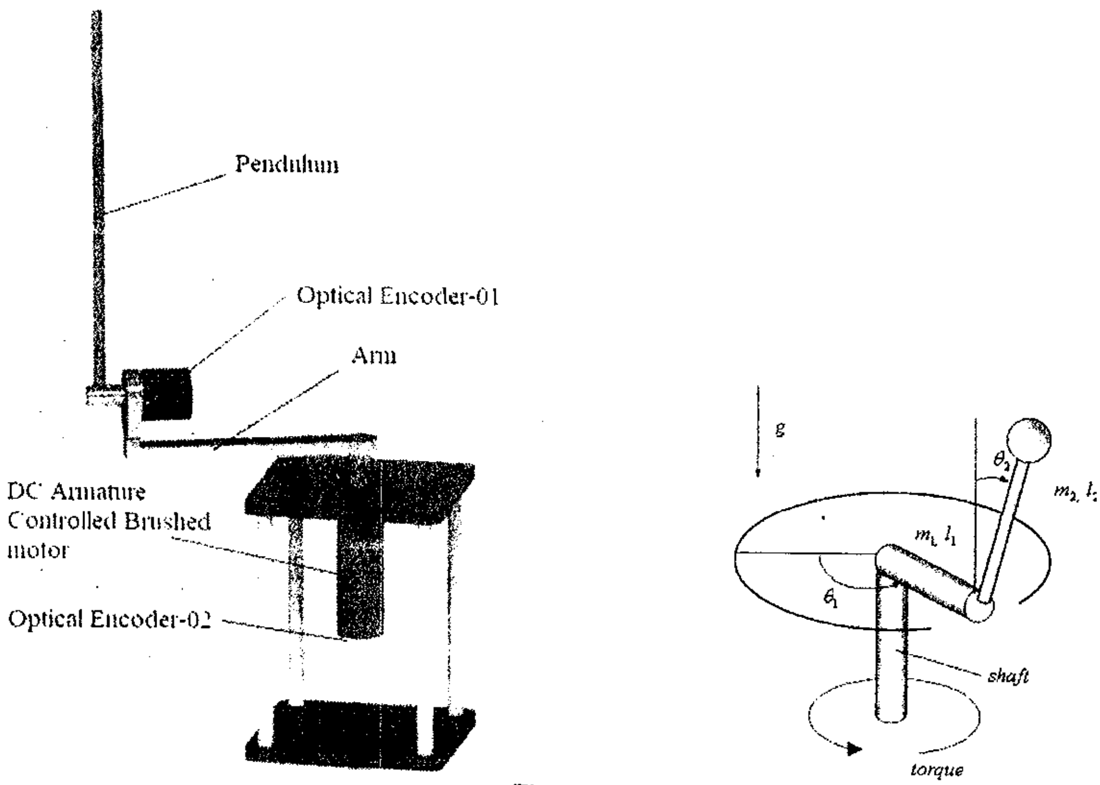
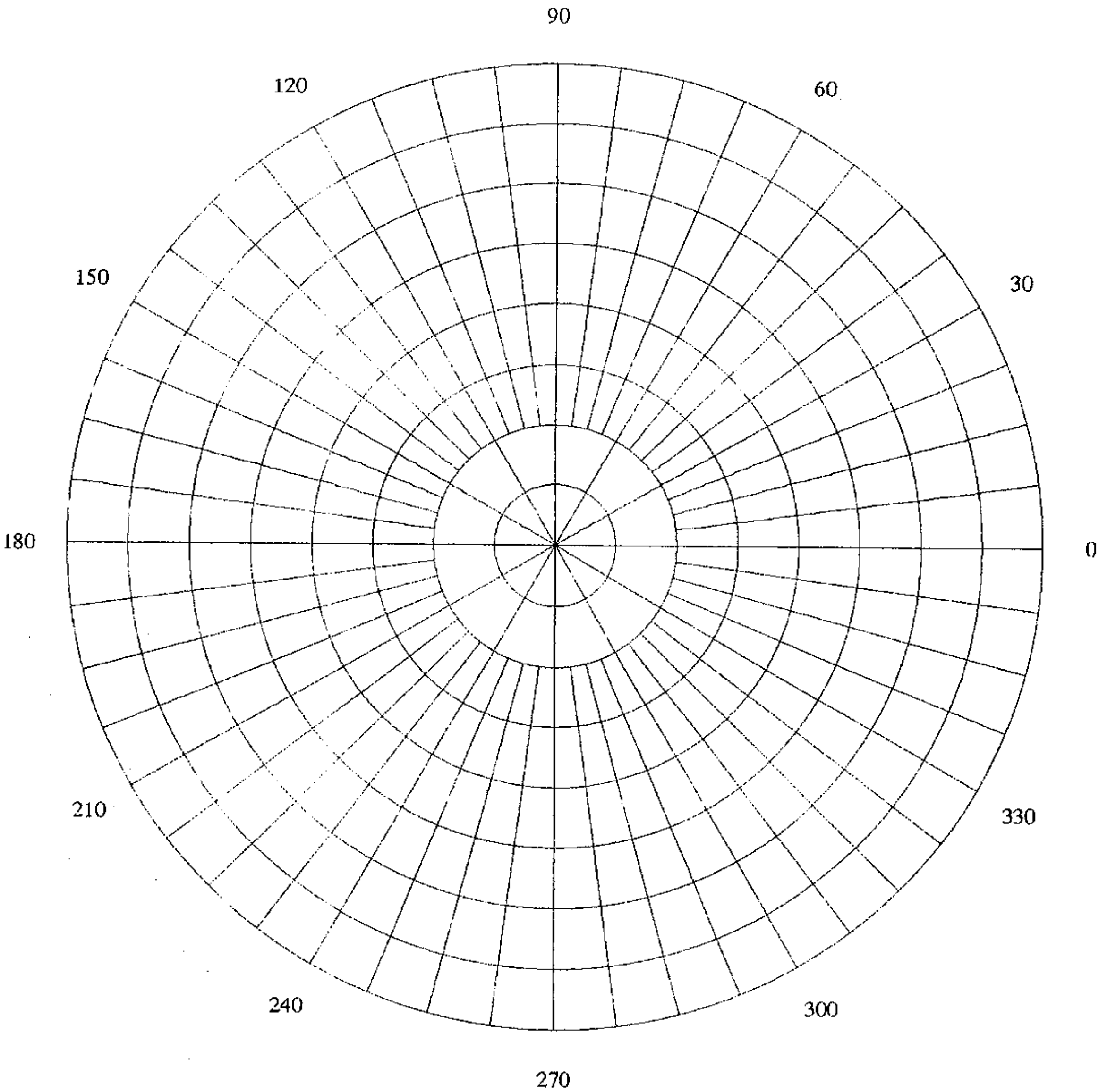


Figure Q5

Note: Only use this graph sheet for answering part Q4.a(iv)



Laplace transforms – Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s + a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s + a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s + a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s + a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s - a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s - a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b - a}(e^{-at} - e^{-bt})$	$\frac{1}{(s + a)(s + b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s + a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s + a)^2}$	$e^{-at} \cos \omega t$	$\frac{s + a}{(s + a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s - a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s - a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s - a}{(s - a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s + a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s + a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s + a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		