UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: October 2019

Module Number: IS5301 **Module Name: Numerical Methods**

[Three hours]

[Answer **all questions,** each question carries 14 marks]

Q1. a) The tolerance, *tol,* of the solution in the bisection method is given by

$$tol = \frac{1}{2}(b_n - a_n),$$

where a_n and b_n are the end points of the interval after the n^{th} iteration. The number of interations n that are required for obtaining a solution with a tolerance that is equal to or smaller than a specified tolerance can be determined before the solution is calculated. Show that n is given by

$$n \geq \frac{\log(b-a) - \log(tol)}{\log 2}$$

where a and b are the endpoints of the starting interval and *tol* is a user-specified tolerance.

[4 Marks]

b) The location $\bar{x}$ of the centroid of a segment of a circle is given by:

$$\bar{x} = \frac{2r\sin^3\alpha}{3\,(\alpha - \sin\alpha\cos\alpha)}$$

By assuming $\alpha = \sin\alpha$,

i.) Determine the angle α for which $\bar{x} = \frac{3r}{4}$. First, derive the equation that must be solved.

ii.) Find the minimum number of iterations which requires to determine the root. The stopping criteria of this function is 0.01 and the given interval is [0.1, 1.4].

iii.) Use the bisection method to calculate the root of the function.

[6 Marks]

c) Find the solution of the equation:

$$f(x) = x^3 - e^{-0.5x}$$

by using Newton Raphson's method. Calculate only the first four iterations to four significant figures. Use $x = 1$ as the initial guess of the solution.

[4 Marks]

Q2. a) Write down the third derivative approximation equation for the

i.) Newtons Forward difference

ii.) Newtons Backward difference

iii.) Newtons Divided difference

[3 Marks]

b) The set of the following five data points (x, y) is given

x	y
1	52
2	5
4	-5
5	-40
7	10

i.) Determine the fourth-order polynomial in Newton's form that passes through all the five points. Calculate the coefficients by using a divided difference table.

[2 Marks]

ii.) Use the polynomial obtain in part (i) to determine the interpolated value for $x = 3$.

[2 Marks]

c) Solve the following set of four equations using LU factorization method.

$$4x_1 - 2x_2 - 3x_3 + 6x_4 = 12$$
$$-6x_1 + 7x_2 + 6.5x_3 - 6x_4 = -6.5$$
$$x_1 + 7.5x_2 + 6.25x_3 + 5.5x_4 = 16$$
$$-12x_1 + 22x_2 + 15.5x_3 - x_4 = 17$$

[7 Marks]

Q3. a) Derive the second derivative approximation equation for the central difference.

[2 Marks]

b) The specific heat capacity is an important element in thermodynamics process. When the pressure is constant, the specific heat capacity C_p, equals to the slope of the relationship between specific enthalpy H, and the temperature T, as follows.

$$C_p = \frac{dH}{dT}$$

Enthalpy H, data obtained with respect to the temperature are given as bellow.

$T\ (^0F)$	$H(Btu/lb)$
800	1250
1200	1480
1600	1725
2000	1945
2400	2050

Calculate the specific heat capacity and rate of change at the temperature 1600^0F.

[4 Marks]

c) A Boeing 727-200 airplane of mass $m = 97000\ kg$ lands at a speed of $90\ m/s$ and applies its thrust reversers at $t = 0$. The force F that is applied to the airplane, as it decelerates, is given by $F = -5v^2 - 570000$, where v is the airplane's velocity. Using Newton's second law of motion and flow dynamics, the relationship between the velocity and the position x of the airplane can be written as:

$$mv\frac{dv}{dx} = -5v^2 - 570000$$

where x is the distance measured from the location of the jet at $t = 0$.
Considering five subintervals, determine how far the airplane travels before its speed is reduced to $40\ m/s$ by using the composite trapezoidal method to evaluate the integral resulting from the governing differential equation.

[4 Marks]

d) The equation of the ellipse shown in the figure Q3 (d) is $4x^2 + y^2 = 1$, and its area is $A = \pi/2$. Consequently, the area of the shaded area is:

$$\int_{-0.5}^{0.5} \sqrt{1 - 4x^2}\, dx = \frac{\pi}{4}$$

i.) Evaluate the integral using the two-point Gauss Quadrature formula.

ii.) Find the absolute relative error for part (i) in above.

Refer the following table for the weighting factors and function argument values.

Point	Weight Factors	Function Arguments
2	$C_1 = 1.0000$	$t_1 = -0.5773$
	$C_2 = 1.0000$	$t_2 = 0.5773$
3	$C_1 = 0.5555$	$t_1 = -0.7746$
	$C_2 = 0.8888$	$t_2 = 0.0000$
	$C_3 = 0.5555$	$t_3 = 0.7746$

[4 Marks]

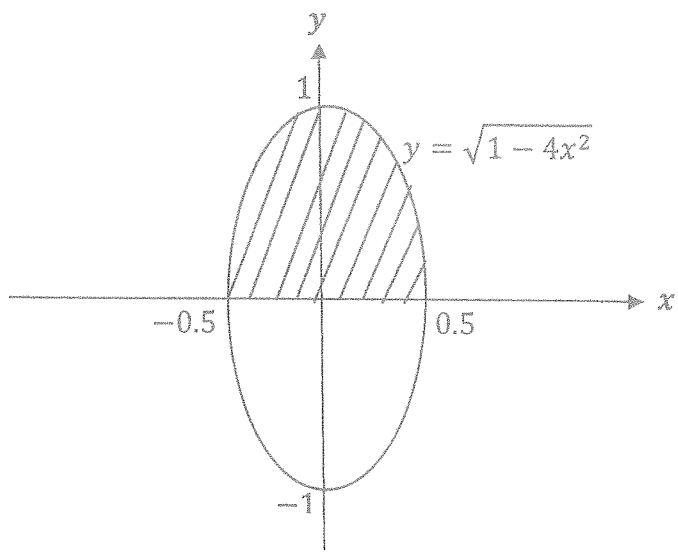

Figure Q3 (d)

Q4. a) Derive Taylor series formula and Picard's Successive Approximations formula starting with $y' = dy/dx = f(x, y)$ with the initial conditions $y = y_0$ for $x = x_0$.

[4 Marks]

b) Consider the initial value problem

$$\frac{dy}{dx} = 3x + \sin(x - y) + 2y, \qquad y(2) = 3.$$

Find $y(2.2)$ with step size $h = 0.2$ to four decimal places by using,

i.) Euler method.

ii.) Second order Runge-Kutta method.

iii.)Compare the two results.

[6 Marks]

c) Using the following ordinary differential equation with given initial conditions and step size of h, show that the error of 4th order Runge-Kutta method is of order 5 $[O(h^5)]$.

$$\frac{dy}{dx} = y, \qquad y(0) = 1$$

[4 Marks]

Q5. a) Find and classify the following partial differential equations as hyperbolic, parabolic, or elliptic $\forall\, x, y \in \mathbb{R}$.

i.) $u_{xx} - 2u_{xy} + u_{yy} + 3u_y - 4u_x = 3x - 2y$

ii.) $(x + 1)u_{xx} - 2(x + 2)u_{xy} + (x + 3)u_{yy} = \cos(x - 2y)$

iii.) $u_{xx} + 4u_{xy} + (x^2 + 4y^2)u_{yy} = \sin(x + y)$

[4 Marks]

b) Use Crank-Nicolson method to solve the partial differential equation,

$$3u_{xx} - u_t = 0 \text{, for } 0 \le x \le 1 \text{ and } 0 \le t \le 0.4$$

with the initial conditions,

$$u(x, 0) = x - x^2$$

and the boundary conditions,

$$u(0, t) = 0$$

$$u(1, t) = t.$$

Use, $h = 0.2$ and $k = 0.2$, where h and k are step sizes along x and t axes respectively.

[8 Marks]

c) Briefly explain the procedure of finite difference solution technique for solving the partial differential equation $u_{xx} + u_{yy} = 0$ highlighting any differences with the solution technique used in part (b).

[2 Marks]