University of Ruhuna - Faculty of Medicine

Allied Health Science Degree Programme
First B. Pharm. Part I Examination - June 2015
PH1152 : Mathematics (SEQ)

Time: Two (02) Hours

Each question carries equal marks

Instructions:

- Answer all questions.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.

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1. a) Let z be a complex number of the form $x+iy$, where x, y are real numbers and i is the imaginary unit.

(i) Write down the complex conjugate z^* of z.

(ii) Show that z^*z and z^*+z are always real.

Determine the value of $\frac{3+5i}{3-2i}$.

b) Expand $(x^2+iy^2)^4$ using the binomial theorem. Here i is the imaginary unit.

c) The number of bacteria N present in a sample is given by $N = 800e^{0.2t}$, where time t is in seconds. Find

(i) the initial number of bacteria.

(ii) the time when the number of bacteria reaches 10^4.
You may use that $\ln 12.5 = 2.52$.

d) Using the formulae for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$, prove that

$$\tan(\alpha+\beta) = \frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}.$$

Hence, deduce that

$$\tan\left(\theta+\frac{\pi}{6}\right) = \frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}.$$

Continued.

2. a) Find the following limits:

(i) $\lim_{x\to -2}\dfrac{x^3+2x^2-1}{5-3x}$,

(ii) $\lim_{x\to 0}\dfrac{(a+x)^3-a^3}{x}$,

(iii) $\lim_{x\to 2}\dfrac{x^4-16}{x-2}$.

b) Differentiate the function $y=3x^2+2x$ with respect to x from the first principles.

c) Differentiate the following functions with respect to x:

(i) $\dfrac{2}{9}\tan\dfrac{3x}{2}-\dfrac{3}{4}\cos 8x$,

(ii) $e^{6x}\ln 6x$,

(iii) $\dfrac{\sin 2x}{\sin 5x}$.

d) A crve has the equation $y=2x^3-7x^2+4x+4$. Find the turning points of the curve and determine their nature using the second derivative $\dfrac{d^2y}{dx^2}$.

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3. a) A three variable function is given by

$$z(a,b,c)=a^4b^2c+2ab+3c+7.$$

(i) Find the partial derivatives

$$\frac{\partial z}{\partial a},\ \frac{\partial z}{\partial b},\ \text{and}\ \frac{\partial z}{\partial c}.$$

(ii) Show that the total differential dz of z at the point $(1,3,5)$ is given by

$$dz=186\,da+32\,db+12\,dc.$$

b) Use integration by parts to evaluate

$$\int x\cos x\,dx.$$

Hence, show that

$$\int x^2\sin x\,dx=-x^2\cos x+2x\sin x+2\cos x+C,$$

where C is an arbitrary constant.

(Hint : You may use that $\int u\,dv=uv-\int v\,du$).

Continued.

4. a) Show that

$$\int \frac{1}{(x-1)(x+2)}dx = \frac{1}{3}\ln\frac{x-1}{x+2}+C,$$

where C is an arbitrary constant.

b) (i) Evaluate

$$\int_0^{2\pi} \cos\theta d\theta.$$

(ii) If

$$\int_0^{2\pi} A(\sin\theta+\cos\theta)d\theta = 1,$$

find A.

c) The gradient of a curve of the form $y = f(x)$ is given by

$$\frac{dy}{dx} = 2(1-x).$$

If $(2,0)$ is a point on the curve, find the equation of the curve.

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