University of Ruhuna
Faculty of Allied Health Sciences
Department of Pharmacy
First B. Pharm. Part I Examination - July 2018

PH1152 : Mathematics (SEQ)

Time: Two (02) Hours

Each question carries equal marks

Instructions:

- Answer all questions.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.

1. a) Find the following limits:

   (i) $\lim_{t\to -2} \dfrac{t^3+8}{t+2}$,

   (ii) $\lim_{u\to 0} \dfrac{\sqrt{u^2+4}-2}{u^2}$.

   b) On a warm day in a garden, water in a bird bath evaporates in such a way that the volume, V *ml*, at time t *hours* is given by

   $$V = \frac{60t+2}{3t},\ t>0.$$

   (i) Find $\dfrac{dV}{dt}$ using **first principles** and show that it is negative.

   (ii) At what rate is the water evaporating after 2 hours?

   (iii) Sketch the graph of $V = \dfrac{60t+2}{3t}$ for $t \in [1/3, 2]$.

   (iv) Calcultae the gradient of the chord joining the endpoints of the graph for $t \in [1/3, 2]$ and explain what the value of this gradient measures.

   c) Differentiate the following functions with respect to x:

   (i) $h(x) = e^{2x+1}\tan(2x)$,

   (ii) $g(x) = \dfrac{1+\cos x}{1-\cos x}$.

2. A veterinarian has administered a painkiller by injection to a sick horse. The concentration of painkiller in the blood, c mg/ℓ, can be defined by the rule

   $$c = \frac{3t}{4+t^2},$$

   where t is the number of hours since the medication was administered.

(i) Find $\frac{dc}{dt}$.

(ii) What is the maximum concentration of painkiller in the blood, and at what time is this achieved?

(iii) The effect of the painkiller is considerably reduced once the concentration falls below $0.5\,mg/\ell$, when a second dose needs to be given to the horse. When does this occur?

(iv) Find the rate of change of concentration of painkiller in the blood after one hour. Give your answer correct to 2 decimal places.

(v) When is the rate of change of concentration of painkiller in the blood equal to $-0.06\,mg/\ell/hour$? Give your answer correct to 2 decimal places.

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3. a) Show that the function

$$f(x,y) = 30x^{1/2}y^{3/2} - 2\frac{x^3}{y}$$

is homogeneous of degree 2 and satisfies the Euler's theorem.

b) Using the substitution $u = t^2 + 4$, evaluate

$$\int \frac{t}{\sqrt{t^2+4}}\,dt.$$

c) Use **integration by parts formula** to show that

$$\int x^2 \ln x\,dx = \frac{1}{9}x^3(3\ln x - 1) + C;$$

where C is an arbitrary constant.

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4. a) The average rate of increase, in $cm/month$, in the length of a baby boy from birth until age 36 months is given by the rule

$$\frac{dL}{dt} = \frac{4}{\sqrt{t}},$$

where t is the time in months since birth and L is the length in centimeters. Find the average total increase in length of a baby boy from 6 months of age until 36 months of age. Give your answer correct to 1 decimal place.

b) Show, by the **method of separation of variables**, that the solution of the differential equation

$$\frac{dy}{dx} = y^2 - e^{3x}y^2$$

can be written as

$$y = \frac{1}{-x + \frac{1}{3}e^{3x} + C};$$

where C is an arbitrary constant.

Given $y = 1$ when $x = 0$, show that $C = 2/3$ and write down the solution in its simplest form.

c) Test the differential equation

$$(2xy + \cos y)\,dx + (x^2 - x\sin y - 2y)\,dy = 0$$

for exactness. If it is exact, then find its solution.

Last Page