University of Ruhuna

Faculty of Allied Health Sciences

Department of Pharmacy

First B. Pharm. Part I Examination - October 2019

PH1152 : Mathematics (SEQ)

Time: Two (02) Hours

Each question carries equal marks. Calculators will be provided.

Instructions:

- Answer all questions.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.

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1. a) Find the following limits:

(i) $\lim_{x \to 4} \frac{x^2 - 16}{x - 4}$

(ii) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$

(iii) $\lim_{x \to 1} \frac{x^2 - 3x + 2}{1 - x}$

b) Differentiate the function $y = 3x^2 - 2x$ with respect to x using **first principles**.

c) Differentiate the following functions with respect to x and give your answers in the simplest form:

(i) $f(x) = 4\sqrt{x} - \frac{6}{\sqrt[3]{x^2}}$

(ii) $g(x) = \tan(x^2 + \sin x)$

(iii) $h(x) = \frac{1 + \ln x}{1 - \ln x}$

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2. a) Find the point at which the tangent to the curve $y = \sqrt{4x - 3} - 1$ has its gradient $2/3$.

b) Consider the function $y = 2x^3 + 3x^2 - 12x + 17$.

(i) Find the turning points of this function.

(ii) Classify the above turning points as maxima or minima using the second derivative $\frac{d^2y}{dx^2}$.

c) Malignant tumours respond to radiation therapy and chemotherapy. Consider a medical experiment in which mice with tumours are given a chemotherapeutic drug. At the time of the drug being administered, the average tumour size is about $0.5 cm^3$. The tumour volume $V(t)$ after t days is modelled by

$$V(t) = 0.005 e^{0.24t} + 0.495 e^{-0.12t}, \text{ for } 0 \leq t \leq 18.$$

(i) Find $V'(t)$ and $V''(t)$.

(ii) Show that the function $V(t)$ has a minimum in $0 \leq t \leq 18$.

(iii) Show that the minimum point in part (ii) occurs when $t \approx 10.84$ days and find the volume of the tumour at this instant.

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3. a) Consider the van der Walls equation

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT,$$

where P, V, T, R have usual meanings and a, b are constants.

Find first partial derivatives of P with respect to T and V.

b) Let $f = \ln(e^{2x} + e^{2y})$. Show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = 2$.

c) Consider the function $g(T, V) = 2V^3T + 2T^3V$.

(i) Find the total differential dg when $V = 2$ and $T = 10$.

(ii) Show that $\frac{\partial}{\partial T}\left(\frac{\partial g}{\partial V}\right) = \frac{\partial}{\partial V}\left(\frac{\partial g}{\partial T}\right)$.

d) Show that the function $f(T, V) = TV^{\gamma - 1}$, where γ is a constant, is a homogeneous function of degree γ. Does this function satisfy Euler's theorem? Justify your answer.

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4. a) Using the substitution $u = x^2 + 1$, evaluate

$$\int 2x\sqrt{x^2 + 1}\, dx.$$

b) Writing $\sin^2 x$ as $\frac{1}{2}(1 - \cos 2x)$, show that

$$\int_0^{\pi/4} \sin^2 x\, dx = \frac{\pi - 2}{8}.$$

c) Use **integration by parts formula** to show that

$$\int x \ln x\, dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C;$$

where C is an arbitrary constant.

d) The gradient of a curve of the form $y = f(x)$ is given by

$$\frac{dy}{dx} = \frac{8}{(5 - 2x)^2}.$$

Show that the equation of the curve is given by

$$y = \frac{4}{5 - 2x} + C;$$

where C is an arbitrary constant.

If this curve passes through the point $(2, 7)$, find the value of C.
