UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: November 2019

Module Number: ME 3305 **Module Name: Modelling and Controlling of Dynamic Systems**

[Three Hours]

[Answer **all questions**, each question carries twelve marks]

A partial table of Laplace transformation pairs is given on page 7. You may make additional assumptions where necessary, but clearly state them in your answers.

Q1 a) Determine the constants α, β, y_0 and $\dot{y}_0$ so that $Y(s) = \frac{s}{(s+1)^2}$ is the Laplace transform of the solution to the initial value problem,

$\ddot{y} + \alpha \dot{y} + \beta y = 0,\ y(0) = y_0,\ \dot{y}(0) = \dot{y}_0$

[3.0 Marks]

b) Consider the *RC* electrical network shown in Figure Q1 (b). Find the output response; $v_o(t)$ in terms of homogeneous response and exogenous response for a unit-step input voltage, $v_i(t)$.

[3.0 Marks]

c) A 16 *kg* mass is attached to a spring with a spring constant of 2 Nm^{-1}. The damping effect is negligible. The mass is released at $t=0$ from the rest at 3 *m* below the equilibrium position. When $t = 2\pi$ seconds, the mass is struck with a hammer, providing an impulse of 4 *Ns*. This situation is modeled by the initial value problem,

$\frac{16}{32}\ddot{y}(t) + 2y(t) = 4\delta(t - 2\pi);\quad y(0) = 3,\ \dot{y}(0) = 0.$

Write a MATLAB code to simulate the displacement function *y(t)*, velocity function $\dot{y}(t)$, and acceleration function $\ddot{y}(t)$, of the mass during $t=0$ to $t=3s$ with the help of 'Forward Euler' numerical method in time domain simulation.

[6.0 Marks]

Q2 a) A motor cycle shock-absorber (Spring-Mass-Damper System) is shown in Figure Q2(a). Mass, spring constant, and, damper coefficient are denoted by $m=1$ *kg*, $k=2$ *N/cm* and $b=3$ *Ns/cm* respectively. (Assume zero initial conditions and zero gravity)

i) Use Newton's laws of motion and derive the transfer function $G(s) = Y(s)/F(s)$, where *Y(s)* and *F(s)* are Laplace Transforms of displacement *y(t)* and external disturbance force *f(t)* (due to bumper on the road), respectively.

ii) Find poles of the system and comment on the stability.

iii) Determine the response *Y(s)* in Laplace domain for a step disturbance force, $f(t)=5u(t)$, where $u(t)$ is the unit step.

Q2 is continued to next page…

iv) Determine the damping coefficient for critically damped response.

v) Draw the approximate sketches of simulating the system for
   a. underdamped,
   b. critically damped,
   c. undamped,
   d. Overdamped conditions, assuming suitable damping coefficients.

vi) Use 'Final Value Theorem' to determine the steady-state value of the output response.

[9.0 Marks]

b) A unit step response of an open loop plant is shown in the Figure Q2(b).
   i) Obtain the DC gain (DCG) of the plant.
   ii) Propose a method to obtain the above response with unity DC gain.

[3.0 Marks]

Q3 a) Figure Q3(a) shows a typical control system. Here the gain K is assumed to be positive. It is observed that for small or large values of K the system is overdamped and, for medium values of K it is underdamped.
   i) Locate the open-loop poles and zeros on the complex plane.
   ii) Determine number of asymptotes and asymptote angle(s).
   iii) Determine the breakaway and break-in points and corresponding feedback gain values of them.
   iv) Sketch the Root Locus.
   v) Determine a sufficient number of points that satisfy the angle condition.

[8.0 Marks]

b) 'Root Locus Design method can be used to locate poles at some desired locations. However, it is not possible to locate poles arbitrarily'. Briefly explain the above statements giving reasons.

[1.0 Mark]

c) The above control system is approximated to a second order system eliminating the existing two zeros. If the plant must provide 5% peak overshoot and 2 s settling time, show that the desired poles are located outside the **Root Locus of the new system**.

[3.0 Marks]

Useful: For under-damped generic second order systems, The transfer function

$G(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, Peak overshoot $PO = e^{\frac{-\varsigma\pi}{\sqrt{1-\varsigma^2}}}$, Settling time $t_s = \dfrac{4.6}{\varsigma\omega_n}$.

Q4 a) A magnetically suspended ball is shown in the Figure Q4(a). The current through the coils induces a magnetic force which can balance the force of gravity and cause the ball (which is made of a soft-magnetic material) to be suspended in mid-air. The equations for the system are given by,

$$m\frac{d^2h}{dt^2} = mg - \frac{Ki^2}{h};$$

$$v = L\frac{di}{dt} + iR;$$

where h is the effective air gap, i is the current through the coil of electromagnet, v is the applied voltage, m is the mass of the ball, g is the acceleration due to gravity, L is the inductance, R is the resistance, and K is a coefficient that determines the magnetic force exerted on the ball. The values are chosen as $m = 0.05$ kg, $K = 0.0001$, $L = 0.01$ H, $R = 1$ Ohm, $g = 9.81$ ms^{-2}. The system is at equilibrium (the ball is suspended in mid-air) whenever $h = Ki^2/mg$ (at which point $dh/dt = 0$). Obtain the non-linear state-space model of the suspended system.

[7.0 Marks]

b) Above nonlinear state space model in the part a) was linearized about its equilibrium point. The linearized system is presented in the form of the state equation,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -10 & -9 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}[u],$$

$$[\theta] = [1\ 0\ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}.$$

Determine the eigenvalues and hence comment on the stability of the suspension system.

[5.0 Marks]

Q5 a) Figure Q5(a) graphically represents three stability criteria of Lyapunov Stability of an autonomous nonlinear dynamical system. The trajectories start at the initial state $x(0)$ and converge to the equilibrium point $\overline{X}$. X_1 and X_2 represent state variables and, δ, γ, ε are noted for the region of attractions.

i) If $\overline{X}=0$, identify A, B, C figures for stable, asymptotically stable, and, unstable situations.

ii) Briefly explain the concept of "exponential stability" based on the above stability criteria.

[3.0 Marks]

b) Briefly explain how Lyapunov's first method (indirect method) can be used to express the stability of non-linear time-invariant systems.

[1.0 Mark]

c) Determine the stability of the equilibrium of the mechanical system at the origin, $m\ddot{y}(t) + b\dot{y}(t) + k_1 y(t) + k_2 y(t)^3 = f(t)$ using Lyapunov's 2nd method. m, b, k_1, k_2 represent constants.

Hint: Use Lyapunov's linearized technique to investigate the equilibrium with $f(t) = 0$.

[4.0 Marks]

Q5 is continued to next page…

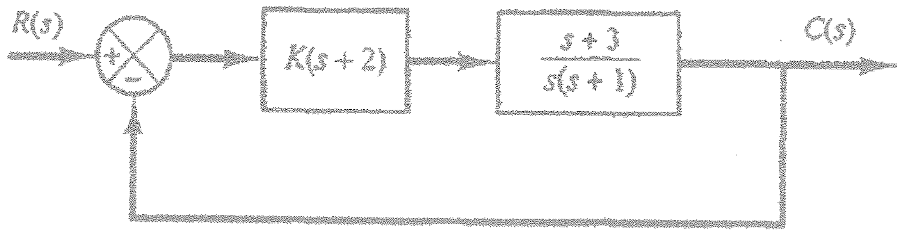

Figure Q3(a): A Control System with a Front Controller

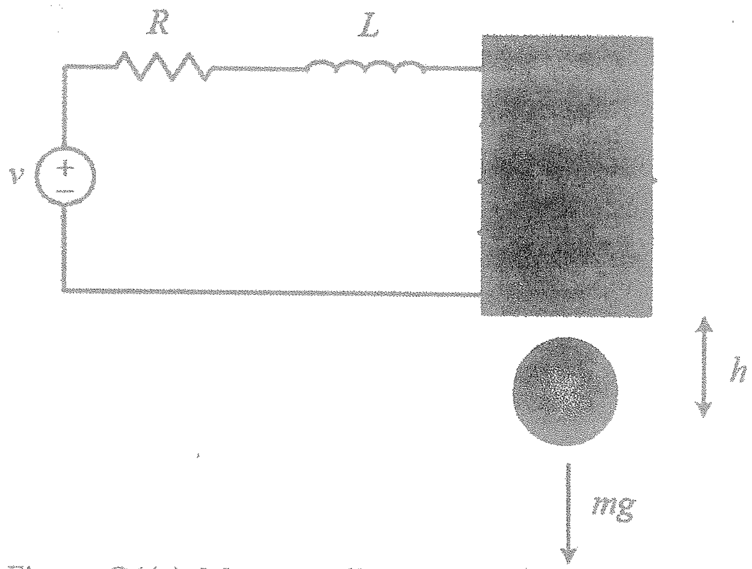

Figure Q4(a): Magnetically Suspended Ball System

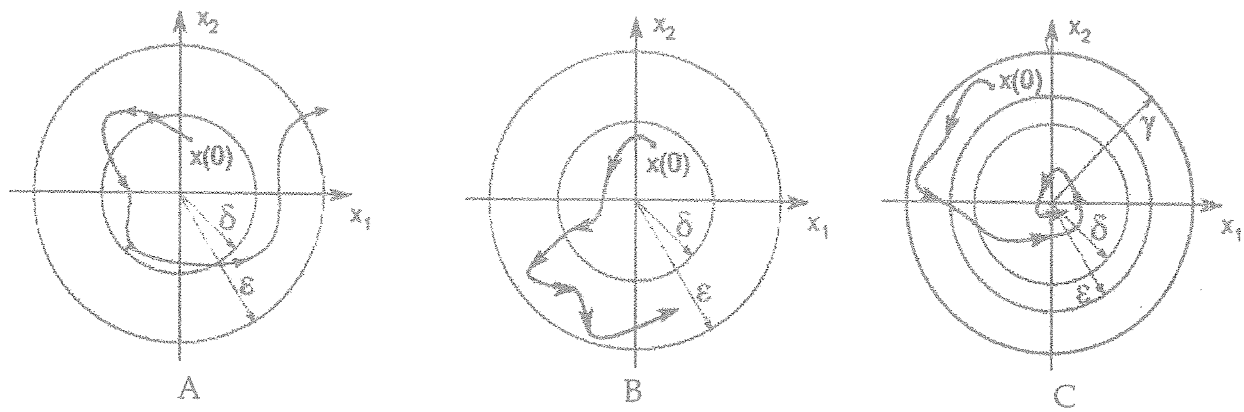

Figure Q5(a): Lyapunov's Stability of an Equilibrium State

Table of Laplace transform pairs

$f(t)$	$F(s)$
step	$\frac{1}{s}$
ramp, t	$\frac{1}{s^2}$
impulse	1
dirac delta function, $\delta(t-c); c \geq 0$	e^{-cs}
$h(t-a)$	$\frac{e^{-as}}{s}$
$h(t-a)\ g(t-a)$	$e^{-as}\,G(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0 \sqrt{1-\zeta^2} t)$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
$1-\frac{\omega_0}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_0 t} \sin(\omega_0 \sqrt{1-\zeta^2} t+\phi)$, Where, $\phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$
$\frac{d(f(t))}{dt}$	$sF(s)-f(0)$
$\frac{d^2(f(t))}{dt^2}$	$s^2F(s)-sf(0)-\dot{f}(0)$
$\int f(t)dt$	$\frac{1}{s}F(s)+\frac{1}{s}\left[\int f(t)dt\right]_{t=0}$
$f(t-\alpha)$	$e^{-\alpha s}F(s)$ with $f(t-\alpha)=0, t \leq \alpha$
$e^{-\alpha t} f(t)$	$F(s+\alpha)$
$t^n f(t);\quad n=1,2,3$	$(-1)^n \frac{d^n F(s)}{ds^n}$
$f(t/a)$	$aF(as)$
Convolution Intergral; $(f_1 * f_2)(t) = \int_0^t f_1(t-\tau) f_2(\tau) d\tau$	$F_1(s)F_2(s)$