UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: August 2018

Module Number: IS5301 **Module Name: Numerical Methods**

[Three hours]

[Answer **all questions,** each question carries 14 marks]

Q1. a) List down the two of each advantages and disadvatages of Newton Raphson method.

[2 Marks]

b) i.) Write down the algorithm for Newton Raphson method.

[3 Marks]

ii.) The following equation occurs in rocket dynamics:

$$m_0[1 - e^{-(v+gt)/v_r}] = u_f t,$$

where m_0 is the mass of the rocket at time $t = 0$, v is the upward velocity at time t seconds, v_r is the relative velocity at which the fuel is ejected, u_f is the fuel consumption rate and g is the acceleration due to gravity ($= 9.8\ m/sec^2$).

Determine t (correct to four decimal places) when $v = 1500 m/\text{sec}, m_0 = 200{,}000$ kg, $v_r = 2500$ m/sec and $u_f = 3000$ kg/sec, using the Newton Raphson method.

[6 Marks]

c) Use the method of Fixed point iteration to find a positive root, between 0 and 1, of the equation $xe^x = 1$. Give your answer to two decimal places.

[3 Marks]

Q2. a) The Lagrangian interpolating polynomial of degree n that passes through $n+1$ data points (x_0,y_0), (x_1,y_1),............, (x_n,y_n) is defined as $P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$.

where, $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$.

Show that $L_i(x_j) = 1$ when $j = i$ and
$L_i(x_j) = 0$ when $j \neq i$.

[2 Marks]

b) Write down the third derivative approximation equation for the

i.) Newtons Forward difference
ii.) Newtons Backward difference
iii.) Newtons Divided difference

[3 Marks]

c) The given table in the below illustrates the upward velocity of a rocket as a function of time.

$t(s)$	$v(t)(m/s)$
0	0
5	217.04
10	352.78
15	507.35
17.5	592.97
20	801.67

i.) Determine the value of the velocity at $t = 11$ seconds with third order polynomial interpolation using Newton's divided difference polynomial method.

[2 Marks]

ii.) Using the third order polynomial interpolant for velocity, find the distance covered by the rocket from $t = 6s$ to $11s$.

[2 Marks]

iii.) Using the third order polynomial interpolant for velocity, find the acceleration of the rocket at $t = 11s$.

[1 Mark]

d) Show that the system

$$\begin{bmatrix} 3+i & 1+2i \\ -3i & 2+i \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 6+2i \\ 1-i \end{bmatrix}$$

can be written as

$$\begin{bmatrix} 3 & 1 & -1 & -2 \\ 0 & 2 & 3 & -1 \\ 1 & 2 & 3 & 1 \\ -3 & 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

where $x_i, y_i \in \mathbb{R}$, $i = 1,2$ and $z_1, z_2 \in \mathbb{C}$.

Solve the above system using Gaussian elimination method and then find z_1 and z_2.

[4 Marks]

Q3. a) Derive Trapezoidal and Simpson's rule formula using a Newtons formula.

[4 Marks]

b) A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines $x = 0$ and $x = 1$, and a curve through the points with the following coordinates:

x	y
0.00	1.0000
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

i.) Apply composite Simpson's rules to estimate the volume of the solid formed, giving the answer to three decimal places.

ii.) Find the absolute relative true error for part (i). (Assume exact value = 2.8294)

[4 Marks]

c) The mechanism of the depolarization process in a fuel cell, an electro-kinetic model for mixed oxygen-methanol current on platinum is developed under the laboratory condition. A developed simplified model of the reaction suggests that it has a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, is given by

$$T = -\int_{1.02\times10^{-5}}^{0.51\times10^{-5}} \left(\frac{6.53x + 4.2015\times10^{-6}}{2.305\times10^{-10}x} \right) dx$$

i.) Use three-point Gauss Quadrature Rule to find the time required for 50 % of the oxygen to be consumed.

ii.) Find the absolute relative true error for part (i) in above (Assume exact value = 157099.7942 s).

Refer the following table for the weighting factors and function argument values.

Point	Weight Factors	Function Arguments
2	$C_1 = 1.0000$	$t_1 = -0.5773$
	$C_2 = 1.0000$	$t_2 = 0.5773$
3	$C_1 = 0.5555$	$t_1 = -0.7746$
	$C_2 = 0.8888$	$t_2 = 0.0000$
	$C_3 = 0.5555$	$t_3 = 0.7746$

[6 Marks]

Q4. a) Derive Taylor series formula and Picard's Successive Approximations formula starting with $y' = dy/dx = f(x,y)$ with the initial conditions $y = y_0$ for $x = x_0$.

[2 Marks]

b) Given, $\frac{dy}{dx} = y^2, \quad y(0) = 1$ find $y(0.1)$ correct to four decimal places by using,

i.) Taylor's method.

ii.) Picard's Successive approximation.

iii.)Find the exact solution and compare the two results you obtained.

[4 Marks]

c) A rectifier-based power supply requires a capacitor to temporarily store power when the rectified waveform from the AC source drops below the target voltage. To properly size this capacitor a first-order ordinary differential equation must be solved. For a particular rectifier-based power supply, with a capacitor of $150\,\mu\text{F}$, the ordinary differential equation to be solved is

$$\frac{dv(t)}{dt} = \frac{1}{150 \times 10^{-6}} \left\{ -0.1 + \max\left(\frac{|18\cos(120\pi(t))| - 2 - v(t)}{0.04}, 0 \right) \right\}$$

$$v(0) = 0$$

Using Euler's method, find the voltage across the capacitor at $t = 0.00004\,\text{s}$. Use step size $h = 0.00002\,\text{s}$.

[4 Marks]

d) Use 4th order Runge-Kutta method to solve,

$$\frac{dy}{dx} = 3x + \frac{y}{2}$$

for $x = 0.1$. Consider the initial conditions as $y(0) = 1$ and the step size of $h = 0.05$.

[4 Marks]

Q5. a) Classify the following partial differential equations as hyperbolic, parabolic, or elliptic.

i.) $8\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} - 3\frac{\partial^2 u}{\partial y^2} = 0$

ii.) $\alpha \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial t} = 0$

iii.) $\frac{\partial^2 u}{\partial x^2} + x^2 \frac{\partial^2 u}{\partial y^2} = 0, \quad x \neq 0$

iv.) $2\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x \partial t} + 3\frac{\partial^2 f}{\partial t^2} + 4\frac{\partial f}{\partial x} + \cos(2t) = 0$

[3 Marks]

b) i.) List advantages and disadvantages of using the implicit method in solving partial differential equations.

ii.) Use Crank-Nicolson method to solve the partial differential equation,

$$\frac{\partial T}{\partial t} = 0.02\frac{\partial^2 T}{\partial x^2} \text{ for } 0 \le x \le 1 \text{ and } 0 \le t \le 1$$

with the initial conditions,

$T(x, 0) = 100x$ for $0 \le x \le 0.6$; $T(x, 0) = 100(1.2 - x)$ for $0.6 < x \le 1$

and the boundary conditions,

$T(0,t) = 0$
$T(1,t) = 20.$

Use, $h = 0.2$ and $k = 0.5$, where h and k are step sizes along x and t axes respectively.

[11 Marks]