University of Ruhuna - Faculty of Medicine

Allied Health Science Degree Programme
First B. Pharm. Part I Examination - July 2016

PH1152 : Mathematics (SEQ)

Time: Two (02) Hours

Each question carries equal marks

Instructions:

- Answer all questions.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.

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1. a) Find all the solutions of the trigonometric equation $2\cos^3 x + \cos^2 x - \cos x = 0$ in the range $0 \leq x \leq 2\pi$. Give your answers in radians.

b) Using the formulae for $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$, prove that $\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$.

Hence deduce that $\tan\left(\theta - \dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}\tan\theta - 1}{\sqrt{3} + \tan\theta}$.

c) Verify the following trigonometric identities.

(i) $\dfrac{\sin\theta}{1 - \cos\theta} - \dfrac{\sin\theta\cos\theta}{1 + \cos\theta} = \operatorname{cosec}\theta\,(1 + \cos^2\theta)$

(ii) $\dfrac{\operatorname{cosec}\theta + \cot\theta}{\tan\theta + \sin\theta} = \cot\theta\operatorname{cosec}\theta$

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2. a) Find the following limits

(i) $\lim_{x \to 3} \dfrac{3x^2 - 8x - 3}{x^2 - 3x}$

(ii) $\lim_{x \to \infty} \dfrac{3x^2 + 2x + 5}{2x^2 + 5x + 7}$

b) If $y = \left(\dfrac{1 + x}{1 - x}\right)^3$, prove that $(1 - x^2)\dfrac{dy}{dx} = 6y$.

c) Consider the function $y = (x + 3)^2(x - 2)^2$.

(i) Find the turning points of this function.

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(ii) Identify the above turning points as maxima, minima or points of inflexion using the second derivative $\frac{d^2y}{dx^2}$.

(iii) Sketch the curve of the above function, clearly showing the locations of turning points.

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3. a) A two variable function is given by

$$f(\alpha,\beta) = \cos 2\alpha \sin\beta.$$

(i) Find the partial derivatives $\left(\frac{\partial f}{\partial \alpha}\right)_\beta$, and $\left(\frac{\partial f}{\partial \beta}\right)_\alpha$.

(ii) Show that the total differential df of f at the point $\left(\frac{\pi}{6},\frac{\pi}{4}\right)$ is given by

$$df = -\frac{\sqrt{6}}{2}\, d\alpha + \frac{\sqrt{2}}{4}\, d\beta.$$

(iii) Prove also that

$$\left[\frac{\partial}{\partial \beta}\left(\frac{\partial f}{\partial \alpha}\right)_\beta\right]_\alpha = \left[\frac{\partial}{\partial \alpha}\left(\frac{\partial f}{\partial \beta}\right)_\alpha\right]_\beta.$$

b) Use integration by parts formula to show that

$$\int x^3 \ln 2x\, dx = \frac{x^4}{4}\ln 2x - \frac{x^4}{16} + C,$$

where C is an arbitrary constant.

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4. a) Using the identity $\cos 2x = 1 - 2\sin^2 x$, show that

$$\int_{\pi/4}^{\pi/2} \sin^2 x\, dx = \frac{\pi+2}{8}.$$

b) The gradient of a curve of the form $y = f(x)$ is given by

$$\frac{dy}{dx} = 3(x+1)(x-2).$$

If the point $\left(2, \frac{2}{3}\right)$ lie on the curve, find the equation of the curve.

c) Test the differential equation

$$(2xy + y^3\cos x)\, dx + (x^2 + 3y^2 \sin x)\, dy = 0$$

for exactness. If it is exact, then find its solution.
