UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 7 Examination in Engineering: August 2018

Module Number: EE7209 **Module Name: Digital Signal Processing**

[Three Hours]

[Answer **all questions**, each question carries **10 marks**]

All symbols have their usual meanings.

Q1 a) The system defined by the input-output expression

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

is called the accumulator system.

i) Rewrite the above expression in the form of a Nth-order linear constant-coefficient difference equation as

$$\sum_{k=0}^{N} a_k \, y[n-k] = \sum_{m=0}^{N} b_m \, x[n-m]$$

ii) If the accumulator is excited by the discrete-time sequence $x[n] = n\,u[n]$, show that its output

$$y[n] = \frac{n^2+n+2}{2} \quad \text{for} \quad n \geq 0$$

when the system has an initial condition $y[-1] = 1$

[4 Marks]

b) Consider the system described by

$$y[n] = \cos(\omega_0 n)\, x[n]$$

i) Explain what this system does.
ii) Is this system linear? Justify your answer.
iii) Is this system time invariant? Justify your answer.
iv) For which conditions does the above system have the BIBO (Bounded Input Bounded Output) stability ?

[6 Marks]

Q2 a) i) What is meant mathematically by the Region of Convergence (ROC) for the z-transform of a discrete-time sequence ?

ii) Explain the possible ROCs for any type of discrete-time sequence which are defined finitely or infinitely.

[4 Marks]

b) Consider the z-transform for the impulse response of a stable LTI (Linear Time-Invariant) given by

$$H(z) = \frac{3}{1 + \frac{1}{3}z^{-1}}$$

Suppose that the input to the system is a unit step sequence i.e., $x[n] = u[n]$.

i) Determine the output $y[n]$ by evaluating the discrete convolution of the input $x[n]$ and the impulse response $h[n]$.

ii) Determine the output $y[n]$ by taking the inverse z-transform of $Y(z)$.

[6 Marks]

Q3 a) Using illustrations, explain the difference between the Fourier transform of a discrete-time sequence (DTFT) and the Discrete Fourier Transform (DFT) of the same discrete-time sequence.

[3 Marks]

b) Suppose that $x_p[n]$ is a periodic sequence with the fundamental period N. Consider the following DFTs (Discrete Fourier Transforms) generated using the different forms of $x_p[n]$.

$$x_p[n] \xleftrightarrow[N]{\text{DFT}} X_1(k)$$

$$x_p[n] \xleftrightarrow[3N]{\text{DFT}} X_3(k)$$

i) What is the relationship between $X_1(k)$ and $X_3(k)$?

ii) Verify the result in part i) using the sequence

$$x_p[n] = \{........, 1, 2, 1, \underset{\uparrow}{2}, 1, 2, 1, 2, ...\}$$

[7 Marks]

Q4 a) Consider the DFT coefficients for a real valued finite length discrete-time sequence given by

$$X(k) = \{2, 1+j, 0, 1-j\} \qquad k = 0, 1, 2, 3$$

Determine the discrete-time sequence using the inverse DFT mathematical expression.

[3 Marks]

b) Compute the 4-point DFT for the obtained discrete-time sequence in part a) using the decimation-in-time Fast Fourier Transform (FFT) algorithm.

<u>Note:</u> Show all the computations made at each intermediary stage.

[5 Marks]

c) Compute the number of real additions and multiplications required for part b).

[2 Marks]

Q5 a) Briefly explain two digital filter design methods (one from each) used in Infinite Impulse Response (IIR) systems and Finite Impulse Response (FIR) systems.

[4 Marks]

b) Consider the LTI (Linear Time-Invariant) system described by the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

i) Draw the signal flow graph to implement the parallel form realization of the above system using a second-order section.

ii) Obtain an alternative parallel form realization with first-order sections by expanding $H(z)$ using all real poles.

[6 Marks]