University of Ruhuna

Bachelor of Science Degree in Fisheries and Marine Sciences
Level I (Semester II) Examination

December 2015

Subject: Mathematics
Course Unit: FSC1b50
Time: Two (02) Hours

Answer **ALL Questions**. Each Question carries equal marks.

1. (a) Find a and b such that

$$a + ib = \frac{1-z}{1+z},$$

where $z = 1 + 2i$ and i is the imaginary unit.

(b) Using the binomial theorem, show that

$$\left(x - \frac{1}{x}\right)^4 = x^4 - 4x^2 + 6 - \frac{4}{x^2} + \frac{1}{x^4}.$$

(c) Solve the following equations for x:

(i) $e^{2x} - 3e^x + 2 = 0$

(ii) $\log_{10}(x+8) + \log_{10}(x-1) = 1$

(d) (i) Write down the expressions for $\cos(\alpha + \beta)$ and $\cos(\alpha - \beta)$.
Prove that

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right].$$

Hence show that

$$\cos 37.5^\circ \cos 7.5^\circ = \frac{\sqrt{2} + \sqrt{3}}{4}.$$

(ii) Verify the following trigonometric identity:

$$\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)} = \frac{\tan\alpha + \tan\beta}{\tan\alpha - \tan\beta}.$$

2. (a) Find the following limits:

(i) $\lim_{h\to 1} \dfrac{h^4 - 1}{h^3 - h}$

(ii) $\lim_{t\to 3} \dfrac{t^2 - 8t + 15}{t^2 + 4t - 21}$

(b) Differentiate the following functions with respect to x:

(i) $y = e^{3x}\sin 3x$

(ii) $y = \dfrac{x^2 + x + 1}{1 - x}$

Continued.

(c) Consider the curve given by $y = x^2 + bx + c$, where b and c are constants. Find the values of b and c such that the straight line $y = 2x$ is tangent to the curve at the point $(2,4)$.

(d) Find the turning points of the curve given by the function $f(x) = 4x^3 + 15x^2 - 18x + 6$ and identify them as maxima or minima using the second derivative $f''(x)$.

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3. (a) Consider the function given by

$$f(x,y) = \sqrt[3]{x^2 + y}.$$

Show that the total differential of f at the point $(2,4)$ is given by

$$df = \frac{1}{12}\left[4dx + dy\right].$$

(b) Show that the function $f(x,y) = 9x^3y + 8x^2y^2 - 6xy^3$ is homogeneous and satisfies the Euler's theorem.

(c) Use integration by parts to evaluate

$$\int x\cos x\, dx.$$

Using integration by parts and the above result, show that

$$\int x^2 \sin x\, dx = -x^2\cos x + 2x\sin x + 2\cos x + C,$$

where C is an arbitrary constant.

(d) Show that

$$\int_1^2 \frac{3t}{t^2 + 4}\, dt = \frac{3}{2}\ln\frac{8}{5}.$$

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4. (a) Find the general solution of the differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x^2 + 1}$$

using the technique of separation of variables.

(b) Test the differential equation

$$(2xy + y^3\cos x)\, dx + (x^2 + 3y^2\sin x)\, dy = 0$$

for exactness. If it is exact, then find its solution.

(c) The following table contains the number of championships the clubs A, B, C, D E, F and G have won over a given period.

Club	A	B	C	D	E	F	G
No. of championships won	9	5	7	x	3	$x+1$	4

If these clubs won 5 championships over the given period of time on average, how many championships did the clubs D and F win over the given period?

Also find the mode, median, range, mean deviation, variance and standard deviation of the number of championships won by these clubs over the given period.

The Periodic Table of the Elements

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 **H** Hydrogen 1.00794																	2 **He** Helium 4.003
3 **Li** Lithium 6.941	4 **Be** Beryllium 9.012182											5 **B** Boron 10.811	6 **C** Carbon 12.0107	7 **N** Nitrogen 14.00674	8 **O** Oxygen 15.9994	9 **F** Fluorine 18.9984032	10 **Ne** Neon 20.1797
11 **Na** Sodium 22.989770	12 **Mg** Magnesium 24.3050											13 **Al** Aluminum 26.981538	14 **Si** Silicon 28.0855	15 **P** Phosphorus 30.973761	16 **S** Sulfur 32.066	17 **Cl** Chlorine 35.4527	18 **Ar** Argon 39.948
19 **K** Potassium 39.0983	20 **Ca** Calcium 40.078	21 **Sc** Scandium 44.955910	22 **Ti** Titanium 47.867	23 **V** Vanadium 50.9415	24 **Cr** Chromium 51.9961	25 **Mn** Manganese 54.938049	26 **Fe** Iron 55.845	27 **Co** Cobalt 58.933200	28 **Ni** Nickel 58.6934	29 **Cu** Copper 63.546	30 **Zn** Zinc 65.39	31 **Ga** Gallium 69.723	32 **Ge** Germanium 72.61	33 **As** Arsenic 74.92160	34 **Se** Selenium 78.96	35 **Br** Bromine 79.904	36 **Kr** Krypton 83.80
37 **Rb** Rubidium 85.4678	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.90585	40 **Zr** Zirconium 91.224	41 **Nb** Niobium 92.90638	42 **Mo** Molybdenum 95.94	43 **Tc** Technetium (98)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.90550	46 **Pd** Palladium 106.42	47 **Ag** Silver 107.8682	48 **Cd** Cadmium 112.411	49 **In** Indium 114.818	50 **Sn** Tin 118.710	51 **Sb** Antimony 121.760	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90447	54 **Xe** Xenon 131.29
55 **Cs** Cesium 132.90545	56 **Ba** Barium 137.327	57 **La** Lanthanum 138.9055	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.9479	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.207	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.217	78 **Pt** Platinum 195.078	79 **Au** Gold 196.96655	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.3833	82 **Pb** Lead 207.2	83 **Bi** Bismuth 208.98038	84 **Po** Polonium (209)	85 **At** Astatine (210)	86 **Rn** Radon (222)
87 **Fr** Francium (223)	88 **Ra** Radium (226)	89 **Ac** Actinium (227)	104 **Rf** Rutherfordium (261)	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (263)	107 **Bh** Bohrium (262)	108 **Hs** Hassium (265)	109 **Mt** Meitnerium (266)	110 (269)	111 (272)	112 (277)	113	114				

Lanthanide series

58	59	60	61	62	63	64	65	66	67	68	69	70	71
Ce Cerium 140.116	**Pr** Praseodymium 140.90765	**Nd** Neodymium 144.24	**Pm** Promethium (145)	**Sm** Samarium 150.36	**Eu** Europium 151.964	**Gd** Gadolinium 157.25	**Tb** Terbium 158.92534	**Dy** Dysprosium 162.50	**Ho** Holmium 164.93032	**Er** Erbium 167.26	**Tm** Thulium 168.93421	**Yb** Ytterbium 173.04	**Lu** Lutetium 174.967

Actinide series

90	91	92	93	94	95	96	97	98	99	100	101	102	103
Th Thorium 232.0381	**Pa** Protactinium 231.03588	**U** Uranium 238.0289	**Np** Neptunium (237)	**Pu** Plutonium (244)	**Am** Americium (243)	**Cm** Curium (247)	**Bk** Berkelium (247)	**Cf** Californium (251)	**Es** Einsteinium (252)	**Fm** Fermium (257)	**Md** Mendelevium (258)	**No** Nobelium (259)	**Lr** Lawrencium (262)