UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: December 2018

Module Number: ME 6302 **Module Name: Automatic Control Engineering**

[Three Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided in the question paper. You may make additional assumptions, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot $= e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1. A second order system is described by the equation (1) where y is the system output and u is the input.

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 7y = 7u \qquad -(1)$$

a) Convert the equation (1) into transfer function form in the Laplace domain. (i.e. Obtain $G(S)$.)

[1.0 Mark]

b) State the assumptions you made during (a) above.

[1.0 Mark]

c) When the input is a unit step, obtain the steady state gain and steady-state error of the system.

[1.0 Mark]

d) Determine the damping ratio, natural frequency and roots of the system.

[2.0 Marks]

e) Sketch S-plane and plot locations of roots (Not necessary to plot to scale). In the same plot, sketch the lines of constant natural frequency and constant damping ratio with clear labels.

[3.0 Marks]

f) Figure Q1 shows the above system $G(S)$ with a controller $C(S)$, Feedback dynamics $H(S)$ and Disturbance $D(S)$. Obtain the expression for $Y(S)$ in terms of $R(S)$, $C(S)$, $D(S)$, $G(S)$ and $H(S)$.

[2.0 Marks]

g) The system shown in Figure Q1 has $H(S) = 1$, and feels disturbance of $\frac{2}{s}$ while subjected to a unit step input. The system has a proportional only controller having a gain of 25. Determine the steady state error of the closed loop system with disturbance.

[2.0 Marks]

Q2. a) An open loop system is described by the following model.

$$G(S) = \frac{2}{S^2+4S+8}$$

i. Draw the closed loop unity negative feedback system with a proportional only controller having a gain of *K*.

[1.0 Mark]

ii. Obtain the characteristic equation for the closed-loop system in (i.) above.

[1.0 Mark]

iii. Find the roots of the characteristic quotation when the value of K is 0, 0.5, 1.0, 1.5, 2.0, and 3.0. Present your answer in a table.

[2.0 Marks]

iv. Draw the root locus of the system (You can use your answer book to draw the root locus indicating important points in *S*-plane (Graph sheets are not necessary)

[2.0 Marks]

v. Briefly explain what is a breakaway point of a root locus plot. Does the root locus draw in (iv.) above has a breakaway point? Explain your answer (use a maximum of five sentences to answer this question).

[2.0 Marks]

b) Figure Q2 shows a root locus of a linear time-invariant system. Obtain open loop zeros and open-loop poles of the system and derive the transfer function.

[4.0 Marks]

Q3. a) A system with a single feedback gain has following open loop transfer function

$$G(S) = \frac{2}{S^2+3S+2}$$

i. The system is controlled by a proportional only controller having negative feedback gain of *K*. Draw the system block diagram and obtain the characteristic equation.

[1.0 Mark]

ii. If 10% overshoot and ±2% settling time under 0.5 s are expected from the plant, obtain the required roots of the characteristic equation.

[3.0 Marks]

iii. Can the system satisfy the required performance of (ii) above by tuning feedback gain *K*? Justify your answer.

[1.0 Mark]

iv. A generic compensator of the form $\frac{S+3}{S+P}$ has been inserted before the system input, to compensate error signal. Draw the system block diagram and clearly show the location of the generic compensator.

[1.0 Mark]

v. Design a compensator to meet the required system performance as stated in (ii) above. What is the type of the compensator designed?

[4.0 Marks]

b) State key characteristics of k_i and k_d gains of a PID controller.

[2.0 Marks]

Q4. a) A plant has the following transfer function.

$$G(s) = \frac{K}{1 + TS}$$

i. Obtain expressions to describe the magnitude and phase of the above system at steady state when subjected to a sinusoidal input.

[2.0 Marks]

ii. Draw the harmonic response diagram of the system and clearly show the expressions describing values of key points.

[2.0 Marks]

b) A transfer function $G(S)$ has the following form. Obtain the steady state time response of the system when the input is sinusoidal. Use the steady state system response and obtain conditions to make the system lead or lag.

$$G(S) = \frac{T_2(1 + T_1S)}{T_1(1 + T_2S)}$$

[3.0 Marks]

c) i. Draw the block diagram of continuous time parallel form PID controller in S domain with a derivative filter having filtering constant of N.

[2.0 Marks]

ii. Mathematically prove that the blocks used in the derivative branch of (i) has continuous time differentiator and low pass filter.

[2.0 Marks]

iii. Briefly explain the advantages of using filtered derivatives in control system implementation for real systems.

[1.0 Mark]

Q5. Figure Q5 shows a typical inverted pendulum system. The expected goal of the system is to keep the pendulum in the vertical position by controlling applied force u. The pendulum is pivoted on the cart as shown.

a) Build a non-linear model to simulate the inverted pendulum system using a block diagram. In your answer, include details about software modules, critical configuration details, locations of sensors, joints and forces. State the expected function of each block.

[3.0 Marks]

b) Indicating system modeled in (a) using a single block, draw the control system block diagram using suitable controller to achieve the vertical position of the pendulum. State the signals of each branch of your model.

[2.0 Marks]

c) The cart is driven by a single DC motor which accepts a maximum of ±3 Ampere current. Draw the block diagram including the motor.

[2.0 Marks]

d) Briefly explain how you would tune the controller of the system modeled in (c) to achieve stable and balanced operation.

[3.0 Marks]

e) A researcher requires to change the position of mass m along the pendulum rod l. However, he does not need to change this at runtime (i.e. while the model is being simulated). Redraw the block diagram of (a) above including the ability to change the position of mass m.

[2.0 Marks]

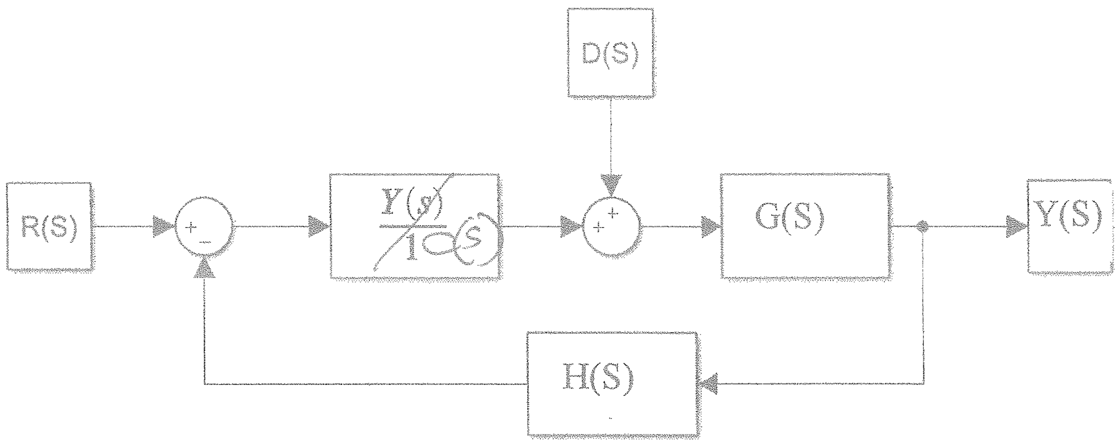

Figure Q1

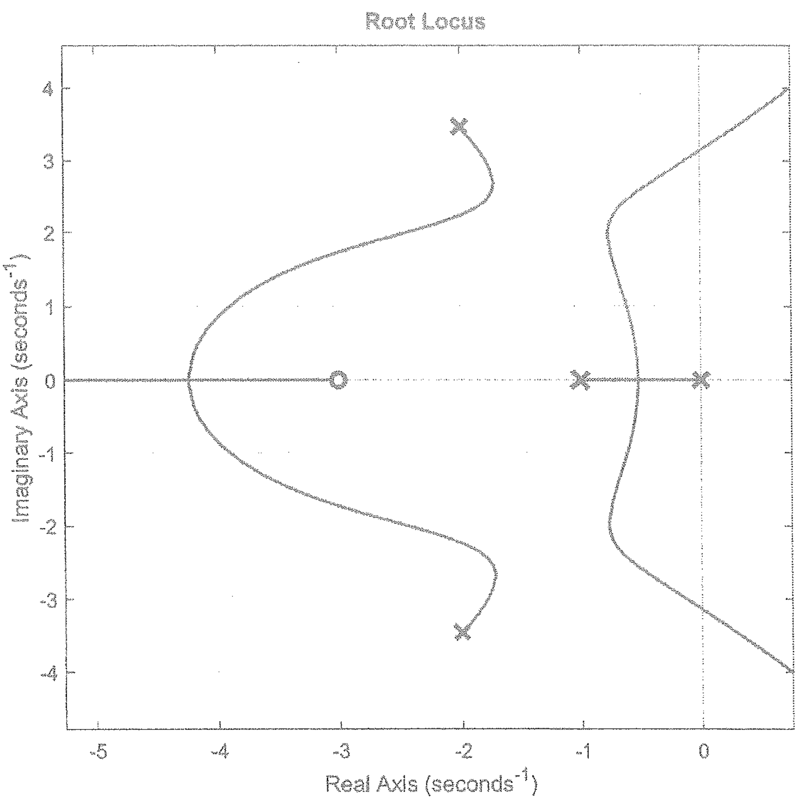

Figure Q2

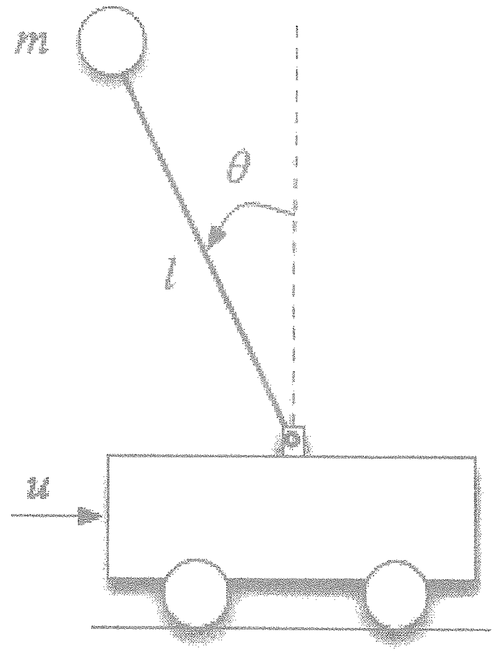

Figure Q5

Laplace transforms – Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!} t^{n-1} e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s} F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		