Spontaneous Symmetry Breaking in Biological Systems

— K. TENNAKONE

Abstract

It is shown that symmetric rate equations describing the interactions between similar species have interesting mathematical properties. The biological implications of their solutions are discussed.

It is well known that the set of equations,

$$\begin{aligned} dN_1/dt &= k_1N_1 = a_1N_1N_2 \\ dN_2/dt &= k_2N_2 = a_2N_2N_1 \end{aligned} \qquad (1)$$

where N_1 and N_2 are the number densities of two interacting populations exhibits competitive selection[1–5] ie, depending on the relative values of the constants k_1, k_2, a_1 and a_2 equations (1) have solutions such that either $N_1 \rightarrow \infty$, $N_2 \rightarrow 0$ as $t \rightarrow \infty$, or $N_2 \rightarrow \infty$, $N_1 \rightarrow 0$ as $t. \rightarrow \infty$. In this note we point out some interesting properties of the symmetric case of the equations (1) where $k_1 = k_2 = k$ and $a_1 = a_2 = a$.

Setting $N_1 = N_2$ in the symmetric equations (1), we obtain the symmetric solution,

$$N_1\ (t) = N_2\ (t) = ka\ (1 + \mathbf{b}\ \mathrm{Exp}\ (\text{-}kt)^{-1}, \qquad (2)$$

where $\mathbf{b}$ = integration constant.

The symmetric equations (1) also have two asymmetric solutions that can be obtained as follows: Subtracting the second equation in (1) from the first and integrating we get,

$$N_1 - N_2 = Ae^{kt} \qquad (3)$$

where A = constant. Eliminating N_2 between (3) and the first equation we get a differential equation for N_1 which can be integrated by looking for solutions of the form $N_1 = f\ (t)\ \mathrm{Exp}\ (kt)$ and the result we obtain for $A > 0$ is

$$\begin{aligned} N_1\ (t) &= Ae^{kt}\ \ (1 - B\ e^{-\ (aA/k)e^{kt}}) \\ N_2\ (t) &= ABe^{kt}\ \ (1 - B^{-1}e^{\ (aA/k)ekt}) \end{aligned} \qquad (4)$$

where B ($0 \geqslant B \geqslant 1$) is a constant. When A is changed to - A we get the other asymmetric solution where the roles of N_1 and N_2 are interchanged, i.e.,

$$N_2(t) = Ae^{kt} \left(1 - B^{-1} e^{-(aA/k)e^{kt}}\right)$$

$$N_1(t) = ABe^{kt} \left(e^{-(aA/k)e^{kt}}\right)\left(1 - B^{-1}e^{-(aA/k)e^{kt}}\right) \tag{5}$$

is obtained. It is seen that in (4) $N_1 \to \infty, N_2 \to 0$ as $t \to \infty$, the reverse happens in (5). Also from (4), $B = (N_2/N_1)_{t \to -\infty}$ and if we set, $B = 1$ the expressions (4) and (5) become two asymmetric solutions of a symmetric set of equations with a symmetric 'initial' condition. In the symmetric solution $N_1 = N_2 = k/a$ as $t \to \infty$, where as in the asymmetric solutions $N_1 = N_2 \to k/a$ as $t \to -\infty$. The plot of N (N_1 or N_2) vs t for the solutions (2), (4) and (5) is indicated in Fig. 1. The phase trajectories plotted with the help of (4) and (5) is shown in Fig. 2. It is seen that the critical point $N_1 = N_2 \to k/a$ is unstable.

The symmetric equations of the type (1) in presence of a inhibitory self-interaction, i.e.,

$$dN_1/dt = kN_1 - aN_1N_2 - bN_1^2$$

$$dN_2/dt = kN_2 - aN_2N_1 - bN_2^2 \tag{6}$$

has a similar property. The same argument shows that if $a > b$, the symmetric solution of (6) is unstable and the stable ones are either $N_1 \to k/b$, $N_2 \to 0$ or $N_1 \to 0$, $N_2 \to k/b$ as $t \to \infty$. It is interesting to note that the strength of the mutual inhibitory interaction (constant a) does not have any influence on equilibrium concentrations of N_1 and N_2. Yet provided $a > b$ both species cannot coexist at the same point.

The above results can be interpretted as follows: In the compétition between two equally compotent biological entities N_1, N_2 once their number densities reach the equilibrium values $N_1 = N_2 = k/a$ an instability develops and the system degenerates into one of the asymmetric states where either N_1 or N_2 survives. Then both species will have an equal chance of survival. Therefore in the spatial distribution any difference in N_1 and N_2 however small due to any fluctuations leads to an expansion of the species in excess, around the point where the fluctuation had occurred. Thus there is a tendency for colonization. The model is also interesting in connection with the cellular differentiation problem. When two species of cells with nearly equal generation times have a mutual inhibitory interaction, the above unstability would lead to colonization, i.e., the 'formation of organs.'

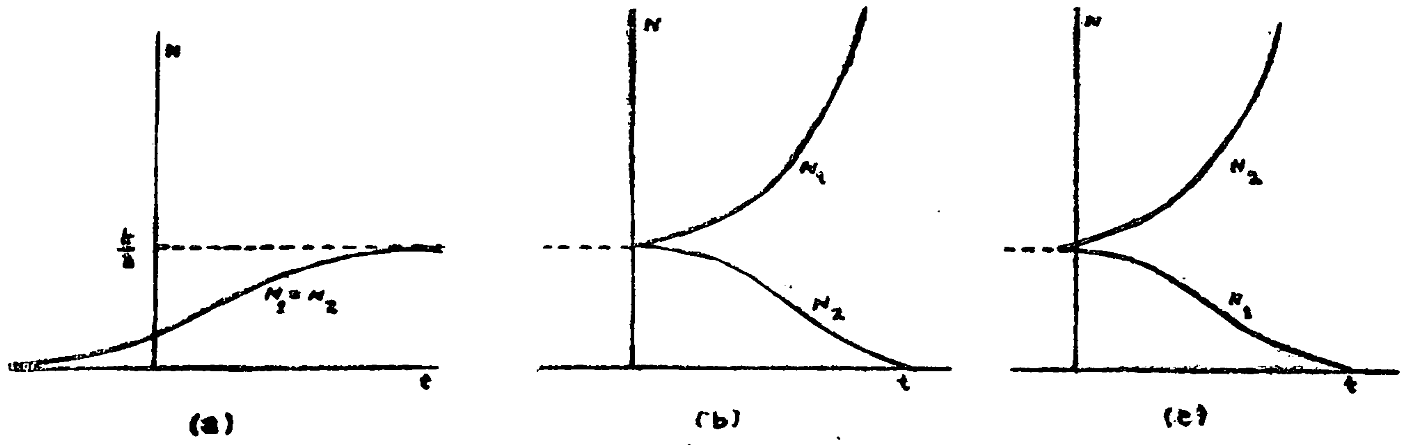

FIG. 1 The plot of N (N_1 or N_2) vs t (a) symmetric solution (b) asymmetric solution, where $N_1 \rightarrow \infty$, $N_2 \rightarrow 0$ as $t \rightarrow \infty$ (c) asymmetric solution where $N_2 \rightarrow \infty$ $N_1 \rightarrow \infty$ 0 as $t \rightarrow \infty$.

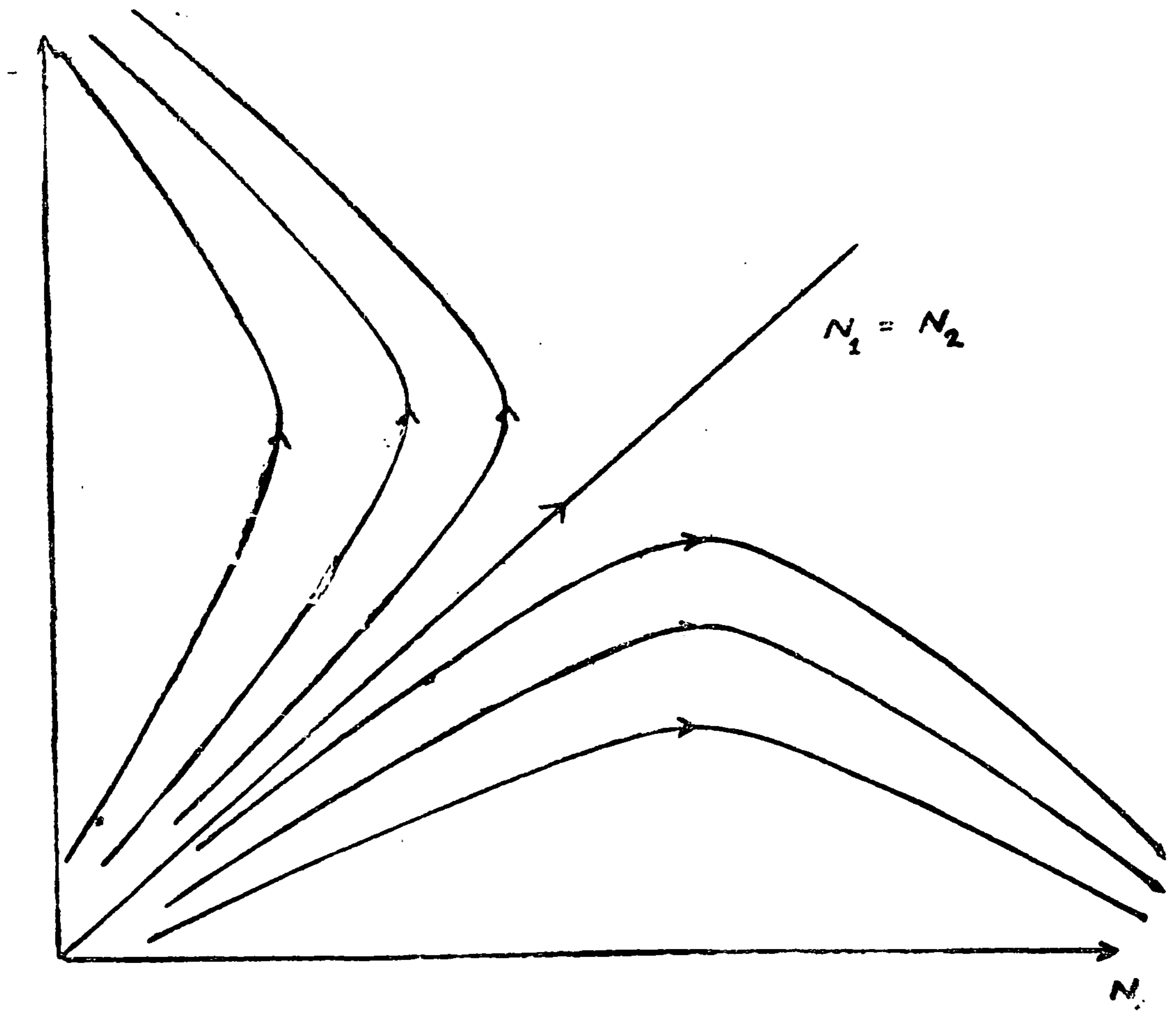

FIG 2. Phase trajectories showing the unstability of the symmetric solution.

We have ignored the diffusion of interacting entities. In the actual situation, diffusion plays an important role in determining the spatio-temporal distribution. When diffusion is included into (1) we obtain the equations

$$\frac{dN_1}{dt} = kN_1 - aN_1 N_2 + bv^2N_1$$

$$\frac{dN_2}{dt} = kN_2 - aN_2N_1 + bv^2N_2 \qquad (7)$$

The analytical solution of the coupled three dimensional partial differential equations (7) is exceedingly difficult. However, it is possible to show that one and two dimensional static solutions of (7) exhibit the phenomenon of spontaneous breaking of the homogeneity of the spatial distribution. In general the stable solutions of equations (7) are not the ones where N_1 (xyz) = N_2 (xyz). Depending on boundary conditions, the stable states correspond to stationary wave-like patterns with regions of high and low concentrations of the two entities 1 and 2. A variegated leaf is probably the simplest example that illustrates the above mechanism. Here N_1 and N_2 are the concentrations of the two types of cells. The two types though very similar in replication rates and intercellular interactions differ because one type produces an accessory pigment. The variegation results because the system is more stable when different components occur as spatially separated colonies, rather than a homogeneous mixture. It is interesting to note that the phenomena discussed above have all the characteristics of a spontaneous breaking of a symmetry[6–8] i.e.,

i. the system has a symmetric unstable state and a set of degenerate asymmetric states

ii. at a critical value of some parameter describing the system, the symmetric solution becomes unstable

iii. unstable solution develops into one of the degenerate asymmetric states.

The development of instabilities and subsequent spontaneous breaking of symmetries could be one of the major factors that has influenced the origin of life[9], the evolution of species and their spatio-temporal distribution.

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Physical Properties of Red Yellow Podzolic Soils under Rubber, Coconut and Cinnamon Cultivation in the Māpalāna Research Farm

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1. Introduction

Destruction of natural plant cover and subsequent cropping lead to a change in the physical condition of the soil and affect its potential for crop growth (Nye and Greenland 1964, Lal 1981). The main effects are on the physical properties such as structure, porosity and compaction resulting in accelerated erosion (Lal *et al* 1980). Soil compaction in its turn has an effect on other water relationships inhibiting infiltration and the moisture holding capacity of a soil (Weerasinghe 1980).

Deterioration of physical conditions of soils often results from improper land development and post-cleaning management systems (Aina P.O. 1983, Lal *et al* 1980). The study of soil physics after long term cultivation of crops therefore deserves close attention.

This paper is aimed at reporting the change of physical conditions such as density, porosity, aggregate stability, mechanical composition and water relationships of red yellow podzolic soil in the University Research Farm, Māpalāna after long term cultivation of rubber, coconut and cinnamon.

The location of study comes under the agro-ecological region of WL_2 (Wet Zone, Low Country) of Sri Lanka. Predominant soils of the region are red yellow podzolic soils, red yellow podzolic soils with strongly mottled sub-soil and low humic grey soils. The mean average rainfall of the area is 1875 mm. and the monthly totals of rainfall follow a bimodal distribution pattern. February and March are relatively rainless and the 75% probability of dry weather comes in January and February.

2. Materials and Methods

Soil pits were made in each location in 3 replicants and soil samples were taken for laboratory analysis. The bulk density of the soil was determined by the core cutter method, true density by the Picnometer method, mechanical composition by the pipette method, and field capacity by the water saturation method; the permanent wilting point was measured by the biological method using triticum sativum as an index plant; the hygroscopic coefficient was measured according to Mitcherlich, soil infitration was determined in the field by the double ring infiltrometer method, and the water stability of soil aggregates by the Yoder's wet seiving technique.

3. Results and discussion

The examination of soil compaction (Table 1) shows that the soils under rubber, coconut and cinnamon are highly compacted compared to those under natural plant cover. The bulk density of the top 30 cm. of soil under plantation crops is about 1.6 g/cm^3 which is probably due to the erosion and absence of soil loosening for many years. High compaction is closely associated with low porosity of soils, and in compacted layers porosity is reduced to 36 - 38%.

The analysis of the mechanical composition of the soils shows that sandy loams are predominant in surface horizons. The clay fraction of the soil shows an increase in subsurface layers. The depth distribution of clay particles as seen in a soil profile has a significant co-relation with bulk density, which shows the downward migration of clay particles through the soil profile and their accumulation in subsurface layers. This relationship is highly pronounced in land under plantation crops.

TABLE 1 — Effect of cinnamon, rubber, and coconut cultivation on the bulk density of red-yellow podzolic soils in the Māpalāna Research Farm (g/cm^3)

Depth	PLANTATION			
	Natural	Rubber	Coconut	Cinnamon
0 — 30	1.32	1.59	1.62	1.61
30 — 60	1.56	1.70	1.61	1.63
60 — 100	1.47	1.80	1.76	1.78

The mean weight diameter (MWD) of the water stable aggregates was high under forest averaging 1.32 mm. for top soils. A slight improvement of water stable aggregates is indicated under rubber. Both the water stable aggregates and dry aggregates were however significantly reduced by cropping coconut and cinnamon (Table 2). These results indicate that the soil was relatively stable under natural plant cover.

The available moisture of the soil (Table 2) is high under natural cover compared to the plantation crops. Apparent reduction of available moisture by about 13% is seen in the soils under rubber, coconut and cinnamon.

TABLE 2 — Soil and water relationship of red-yellow podzolic soils under cinnamon, rubber and coconut plantation and under natural plant cover (depth 0 - 40 cm).

Crop	MWD of aggregates (mm.)		Available moisture in cm.
	dry	wet	
Natural cover	2.66	1.31	26.14
Rubber	2.26	1.80	23.99
Coconut	1.81	0.70	21.43
Cinnamon	1.70	0.60	23.98

The infiltration rate and infiltration capacity are seen to be considerably high under natural plant cover (Fig. 1). 70 cm. of water is infiltered into soil

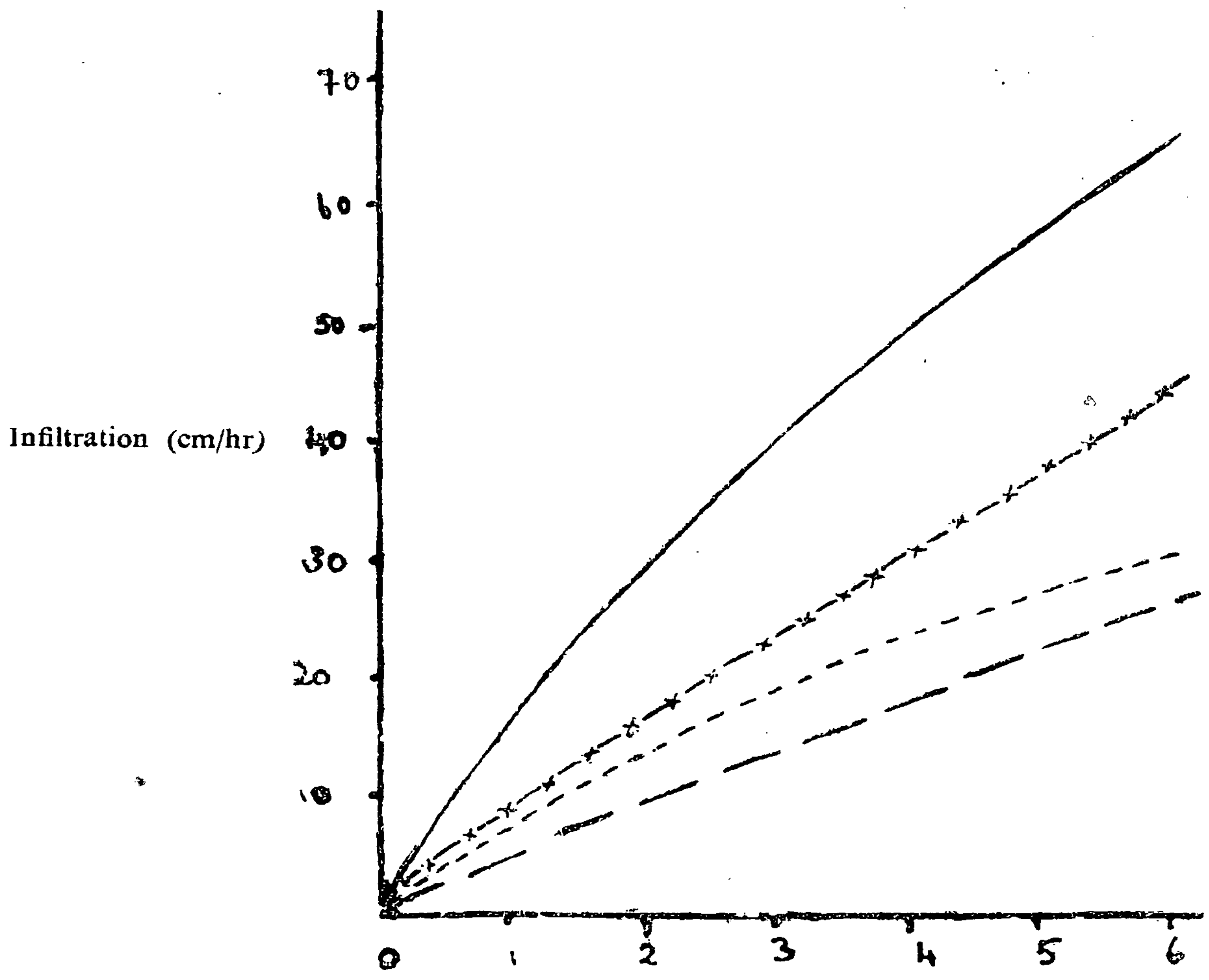

FIG. 1. Cumulative infiltration of red-yellow podzolic soils under, from top to bottom, natural plant cover, coconut, rubber, and cinnamon.

within 6 hrs. The good structural condition and low compaction of the soil under natural cover permit a high infiltration rate. It is significantly less under plantation crops. The reduction of water infiltration is apparently due to changes in the physical conditions of the soil. Cinnamon, rubber, and coconut increase the soil bulk density, and reduce porosity, the combined effect of which would reduce water infiltration. The decrease in infiltration capacity is, however, lower under coconut plantation than under other crops, which is probably due to the well developed root system of the coconut plant.

Conclusions

This study shows that soils deteriorate rapidly following conversion of natural forest lands into crop lands. The deterioration is greatest under cinnamon cultivation due to a relatively high tendency for compaction and aggregate destruction. The water holding capacity and the rate of infiltration of water into the soil is also reduced under rubber, coconut and cinamon plantations as compared to a natural plant cover.

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