

A Method to Compute Unitaries Representing Reducible Minimal Inner Toral Polynomials using Direct Sum

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An inner toral polynomial is a polynomial in two complex variables, $p(z, w) \in \mathbb{C}[z, w]$, such that its zero set is contained in $\mathbb{D}^2 \cup \mathbb{T}^2 \cup \mathbb{E}^2$, where \mathbb{D} , \mathbb{T} , and \mathbb{E} are the open unit disk, unit circle and exterior of the closed unit disk, respectively. We say the bidegree of $p(z, w) \in \mathbb{C}[z, w]$ is (n, m) if p has degree n in z and m in w . An inner toral polynomial p is called a minimal inner toral polynomial if it divides any other polynomial with the same zero set as itself. In the paper Agler, J. & McCarthy, J. (2005) Distinguished Varieties. Acta Math 194, 133-153, the authors proved the existence of unitary matrices representing inner toral polynomials. Specifically, given a minimal inner toral polynomial $p(z, w)$ of bidegree (n, m) , there exists a unitary matrix, written in block form as $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$, such that $\det \begin{pmatrix} A - wI_m & zB \\ C & zD - I_n \end{pmatrix}$ is a constant multiple of $p(z, w)$. Here, blocks A, B, C and D are matrices with complex entries and of sizes $(m \times m)$, $(m \times n)$, $(n \times m)$ and $(n \times n)$ respectively. We call such unitary matrices unitaries representing p . In this work we focused on constructing a method to compute unitaries representing reducible minimal inner toral polynomials using unitaries representing its factors. We prove that if the minimal inner toral polynomial p is a product of s distinct irreducible factors, say, p_1, p_2, \dots, p_s , and if $U_k = \begin{pmatrix} A_k & B_k \\ C_k & D_k \end{pmatrix}$ is a unitary representing p_k for $k = 1, 2, 3, \dots, s$, then the matrix $U = \begin{pmatrix} \bigoplus_D A_k & \bigoplus_D B_k \\ \bigoplus_D C_k & \bigoplus_D D_k \end{pmatrix}$ is a unitary representing p , where $\bigoplus_D A_k = \begin{pmatrix} A_1 & 0 & \dots & 0 \\ 0 & A_2 & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & 0 & 0 & A_s \end{pmatrix}$, the diagonal-wise direct sum of the block matrices A_k 's and $\bigoplus_D B_k, \bigoplus_D C_k$ and $\bigoplus_D D_k$ are defined in similar fashion.

Key words: *Inner toral polynomials, distinguished varieties, block matrices, direct sums, unitaries*

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