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An Efficient Numerical Approximation to Poisson Problem in Two Dimensions

Charuka Dilhara Wickramasinghe

Department of Mathematics, Wayne State University, MI, USA

A numerical solution to the two-dimensional Poisson problem via Finite Element Method is studied in this work. The solution is tested on a squared domain (convex) and an L-Shaped domain (non-convex). The L-Shaped domain is mainly compared with uniform and graded meshes. Poisson equation $-\Delta u = f$ arises in many varied physical scenarios such as heat conduction, electrostatics, Newtonian gravity potential, the motion of an inviscid fluid (Navier Stokes Equation), and the motion of biological organisms in a solution. Although exact solutions to Poisson's equation are known, solutions to such physical models are complicated and difficult to calculate due to the singularities caused by non-smooth geometric boundaries. Numerical methods become necessary to efficiently model solutions of these partial differential equations. Exact solutions to various boundary value problems are still not known. Therefore, we will demonstrate the accuracy of this numerical method by comparing approximate solutions with their projections. A MATLAB program was developed to solve this system with inputs, such as Dirichlet boundary conditions and a nonhomogeneous constant source function f. Piecewise continuous linear finite elements are used to approximate the solution. The convergence rate $r = log_2 \left|\frac{e_{i-1}}{e_i}\right|$ of H^1 and L^2 norm errors are calculated. Here, $e_i = \left\|u_{p_i} - u_{h_{i+1}}\right\|$ is the norm error between the projection values of the i^{th} refinement and the finite element solutions of the $(i+1)^{st}$ refinement. Numerical results indicate that the convergence rate is optimal for the $H^1$and the L^2 norms. Thus, it can be seen that our numerical results agree with a priori error estimates.

Key words: *Finite elements, graded mesh, L-shaped domain, a priori error*

*Corresponding author: gi6036@wayne.edu