UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: December 2018

Module Number: EE8210 **Module Name: Digital Communication**

[Three Hours]

[Answer **all questions**, each question carries **10 marks**]

Q1 a) Briefly explain how orthonormal basis functions are used to simplify the detection of signals in digital communication systems.

[2.0 Marks]

b) A digital communication system that uses two orthonormal basis functions $\phi_1(t)$ and $\phi_2(t)$ are shown in Figure Q1. The three signal vectors s_1, s_2 and s_3 which are in the space spanned by the orthonormal basis functions, are given as

$$s_1 = (\sqrt{2}, 0) \qquad s_2 = \left(-\frac{1}{\sqrt{2}}, \frac{3}{\sqrt{6}}\right) \qquad s_3 = \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{6}}\right).$$

i) Find the values A and B.

[2.0 Marks]

ii) Sketch the waveforms $s_1(t), s_2(t)$ and $s_3(t)$.

[3.0 Marks]

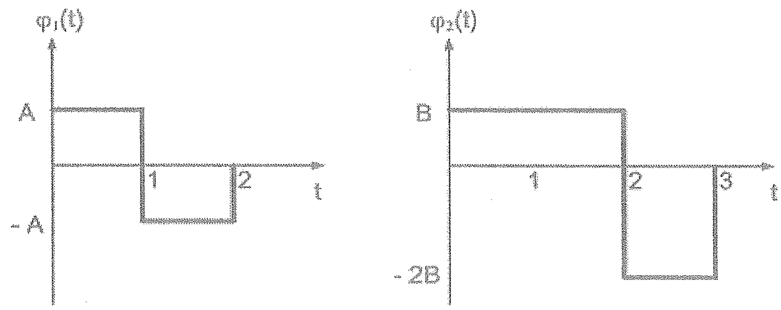

Figure Q1

iii) Assume that you are asked to design a matched filter receiver for this system. Sketch the block diagram of your design, showing the impulse responses of the matched filters.

[3.0 Marks]

Q2 a) When simulating a digital communication system, the following two functions are used.

$$Q(x) = P(X > x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{(-\omega^2/2)} d\omega$$

$$erf(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{(-t^2)} dt$$

Show that $Q(y) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{y}{\sqrt{2}}\right)$ for $y = \sqrt{2}x$

[4.0 Marks]

b) Suppose that the receiver of a digital communication system uses the $X_2 = X_1 + N$ expression where $X_1 \in (0,1)$ with equal probabilities and N is independent of X_1. The mean and the variance of N are 0 and 0.5 respectively. Furthermore, the receiver makes decisions based on the following criteria.

- If $X_2 \le \frac{1}{3}$, then the receiver decides that the transmitter has sent a "0".
- If $X_2 > \frac{1}{3}$, then the receiver decides that the transmitter has sent a "1".

Determine the error probabilities at the receiver in terms of $Q(.)$ function for the following cases.

i) Given that $X_1 = 1$, the receiver decides the transmitter has sent a "0".

ii) Given that $X_1 = 0$, the receiver decides the transmitter has sent a "1".

[6.0 Marks]

Q3 Consider the 4-ary Pulse Amplitude Modulation (PAM) constellation with equi-probable symbols, shown in Figure Q3-1, where the average symbol energy is 4.5 mJ. The non-return to zero pulse shape $p(t)$ shown in Figure Q3-2 is used to modulate the information symbols.

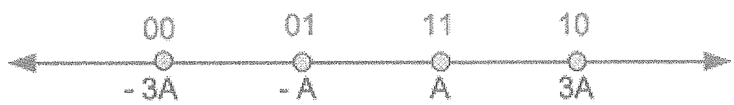

Figure Q3-1

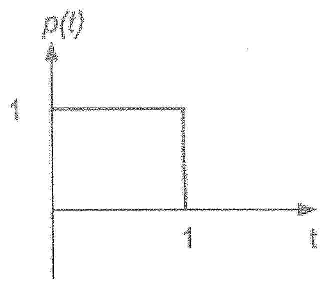

Figure Q3-2

a) Sketch the output waveform of the PAM modulator for the bit sequence 1 0 0 1 0 1 1 0.

[2.0 Marks]

b) Assuming the maximum-likelihood detection rule, sketch the decision boundaries for each symbol.

[2.0 Marks]

c) The output samples of the matched filter receiver of this PAM system are given as $\{+0.071,\ +0.055,\ -0.032,\ 0.101\}$. Determine the transmitted bit sequence.

[2.0 Marks]

d) Obtain an expression for the average symbol error probability for this PAM system, when it is operating in an Additive White Gaussian Noise (AWGN) channel, where the noise has a zero mean and variance σ^2.

[4.0 Marks]

Q4 Consider the three 8-ary Quadrature Amplitude Modulation (QAM) constellations with equi-probable symbols, shown in Figure Q4.

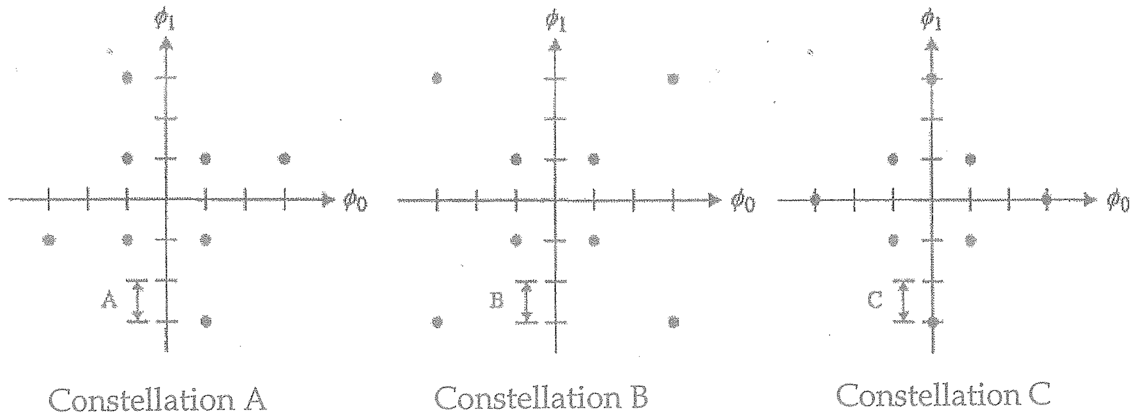

Figure Q4

a) Calculate the average symbol energy of each constellation.

[1.5 Marks]

b) Calculate the minimum distance between any two constellation points, d_{min}, for each constellation.

[1.5 Marks]

c) Calculate the average number of nearest neighbours, N_{min}, for each constellation.

[1.5 Marks]

d) Obtain the average symbol error probability for each constellation using the nearest neighbour approximation.

[3.0 Marks]

e) Assuming all three constellations have equal average symbol energies, identify the constellation with the lowest average symbol error probability. Justify your answer.

[2.5 Marks]

Q5 a) Let X be the outcome of a throw of a fair dice which has six faces marked with 1, 2, 3, 4, 5, 6 respectively. Y is another outcome that depends on X, and Y can be either Even or Odd. To this end, outcome Y is Even, if X is even, and Odd, if X is odd. Calculate

i) $H(X), H(Y)$

[2.0 Marks]

ii) $H(X,Y), H(X|Y), H(Y|X)$

[3.0 Marks]

b) Consider a discrete information source which produces symbols at a rate of 1000 symbols/second. The source alphabet consists of symbols $\{A, B, C, D, E\}$, which have probabilities of occurrence of $P_A = 0.07$, $P_B = 0.7$, $P_C = 0.04$, $P_D = 0.14$ and $P_E = 0.05$.

i) Design a Huffman code for this source.

[3.0 Marks]

ii) Show that your code satisfies the bound for the optimal codeword length.

[1.0 Mark]

iii) Calculate the bit-rate of the Huffman coded system.

[1.0 Mark]