UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: July 2017

Module Number: IS1401 **Module Name: Mathematical Fundamentals for Engineers**

[Three hours]

[Answer **all questions,** each question carries fourteen marks]

Write your answers for **PART-A** and **PART-B** in separate booklets

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PART-A

Q1. a) i.) Show that, if both the sum and product of two non zero complex numbers are real then either the numbers are real or one is the complex conjugate of the other.

ii.) Find the modulus and principal argument of

$$\left(\frac{2+i}{3-i}\right)^2.$$

[2 Marks]

b) i.) Clearly state the De Moivre's Theorem and prove it for positive integers.

ii.) Prove that if n is a positive integer,

$$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1}\cos\frac{n\pi}{6}.$$

[4 Marks]

c) i.) Express, $\sin^7\theta$ as a sum of sines of multiples of θ.

ii.) Find all the roots of $Z = \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^{1/2}$ and graph these roots in the complex plane.

[5 Marks]

d) Find the general value of $\log(1+i)+\log(1-i)$. [3 Marks]

Q2. a) Briefly explain the following types of matrices, and give an example for each type.

i.) Skew symmetric matrix
ii.) Upper triangular matrix
iii.) Hermitian matrix
iv.) Singular matrix

[2 Marks]

b) Determine the values of α, β, γ when

$$\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix} \text{ is orthogonal.}$$

[3 Marks]

c) i.) Let A and B be non-singular matrices of the same order and A^{-1} and B^{-1} are the inverse matrices of A and B respectively. Show that $(AB)^{-1} = B^{-1}A^{-1}$.

ii.) By the method of matrix inversion, solve the following matrix equation.

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x & u \\ y & v \\ z & w \end{pmatrix} = \begin{pmatrix} 9 & 1 \\ 52 & 15 \\ 0 & -1 \end{pmatrix}$$

[5 Marks]

d) Find an equation involving g, h and k that makes the below augmented matrix correspond to a consistent system.

$$\left(\begin{array}{ccc|c} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{array}\right)$$

[4 Marks]

Q3. a) i.) Briefly explain what is meant by 'Unit vector' and 'Position vector'.

ii.) If $\underline{a} = 3\underline{i} - \underline{j} - 4\underline{k}$, $\underline{b} = -2\underline{i} + 4\underline{j} - 3\underline{k}$, $\underline{c} = \underline{i} + 2\underline{j} - \underline{k}$, find the unit vector parallel to $2\underline{a} - 3\underline{b} + 4\underline{c}$.

iii.) Find the position vector of the centroid of a triangle ABC, when position vectors of A, B and C are $\underline{a}$, $\underline{b}$ and $\underline{c}$ respectively.

[5 Marks]

b) A rigid body is spinning with angular velocity 2 radians/sec about an axis OR where position vector of R is $(2\underline{i} - 2\underline{j} + \underline{k})$ and O is the origin. Find the velocity of the point $(3\underline{i} + 2\underline{j} - \underline{k})$ on the body.

[4 Marks]

c) i.) The temperature at any point in space is given by $T = xy+yz+zx$. Determine the derivative of T, (i.e. ∇T), in the direction of the vector$(3\underline{i} - 4\underline{k})$ at the point (1,1,1).

ii.) Given the vector field,

$$\underline{V} = (x^2 - y^2 + 2xz)\underline{i} + (xz - xy + yz)\underline{j} + (x^2 + z^2)\underline{k}.$$

Find *curl* $\underline{V}$.

Show that the vectors given by *curl* $\underline{V}$ at P_0 (1,2,-3) and P_1 (2,3,12) are orthogonal.

[5 Marks]

PART-B

Q4. a) i.) Explain what is meant by the function $f(x)$ is differentiable at a.

ii.) Show that, if $f(x)$ and $g(x)$ are differentiable functions then $(f \circ g)(x)$ is also differentiable and $(f \circ g)' = (f' \circ g).g'$.

[4 Marks]

b) i.) Write down an example for a real function $f(x)$, which satisfies each of the following condition.

I) Limit of $f(x)$ exists for all $x \in \mathbb{R}$, but $f(x)$ is not continuous at $x = 1$.

II) $f(x)$ is continous only for $x \in \mathbb{R} - \mathbb{Z}$.

ii.) Show that the limit at $x = 0$ of the following function does not exist.

$$f(x) = \frac{e^{1/x} - 1}{e^{1/x} + 1}$$

iii.) Discuss the continuity of the following functions.

$$f(x) = \begin{cases} 2x - 1\,; & x < -1 \\ 4x + 1\,; & -1 \leq x < 1 \\ 2x + 1\,; & 1 \leq x \end{cases}$$

[6 Marks]

c) Evaluate the following limits

i.) $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{x \cos x + 2 \sin x}$ ii.) $\displaystyle\lim_{x \to \infty} \frac{\sqrt{(x^2 + 1)}}{2x}$

[4 Marks]

Q5. a) i.) State the Rolle's Theorem

ii.) Show that the equation $2x^3 - 3x^2 + 6x + 4 = 0$ has exactly one real root

[3 Marks]

b) i.) State and prove the Mean Value Theorem

ii.) Use Mean Value Theorem to show that, for $x > 0$

$$\frac{x}{x+1} < \ln(x + 1) < x$$

Deduce that,

$$\frac{x}{e^x} < \ln(x + 1)$$

[6 Marks]

c) i.) State and prove the Euler's Theorem on homogeneous function of two variables.

ii.) Let

$$u = \sin^{-1}\left(\frac{x^3 - y^3}{x + y}\right).$$

Show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

[5 Marks]