UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: July 2017

Module Number: EE3205 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions,** each question carries 10 marks]

All the symbols have their usual meanings.

Q1 a) i) Explain how a continuous-time signal is classified as either a power signal or an energy signal.

ii) Show that the power of $g(t) = C_1\cos(\omega_1 t+\theta_1) + C_2\cos(\omega_2 t+\theta_2)$ is

$$[C_1^2 + C_2^2 + 2C_1C_2\cos(\theta_1-\theta_2)]/2,$$

if $\omega_1 = \omega_2$.

[4.0 Marks]

b) Consider the continuous-time Linear Time-Invariant (LTI) system with the input $x(t)$ and the unit impulse response $h(t)$. Given that $x(t) = e^{2t}u(-t)$ and $h(t) = u(t-3)$, use the convolution integral to determine the output of the system.

[6.0 Marks]

Q2 a) Explain the meaning of determining the Fourier series of a periodic signal $g(t)$ using its exponential Fourier series.

[2.0 Marks]

b) i) Determine the trigonometric Fourier series of the signal $g(t) = t$ to represent $g(t)$ over the interval $(-\pi, \pi)$.

ii) Sketch the line spectrum of the Fourier series of $g(t)$ for all values of t.

[5.0 Marks]

c) Using the Parseval's theorem and the trigonometric Fourier series of $g(t)$ in part b), show that

$$\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right) = \frac{\pi^2}{6}$$

[3.0 Marks]

Q3 a) Explain how it can be obtained the Fourier transform of a non-periodic signal by using the exponential Fourier series of a periodic signal $g(t)$.

[2.0 Marks]

b) Show that the Fourier transform of the triangular pulse $g(t)$ shown in Figure Q3 b) is $G(\omega) = \frac{1}{\omega^2}(e^{j\omega} - j\omega e^{j\omega} - 1)$

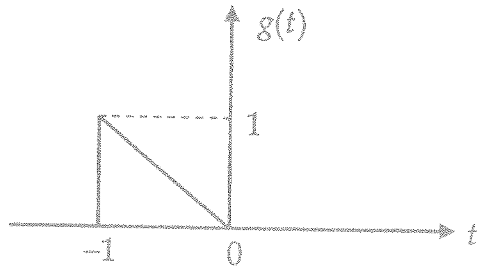

Figure Q3 b)

[5.0 Marks]

c) Using the result in part b), determine the Fourier transform of the aperiodic signal $g_1(t)$ shown in Figure Q3 c).

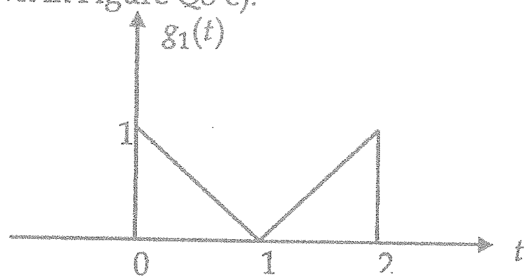

Figure Q3 c)

Hint: Express $g_1(t)$ as a combination of time reversal and shifted versions of $g(t)$ and use time delaying and time scaling properties of the Fourier transform.

[3.0 Marks]

Q4 a) Obtain the relationship between the Laplace transform and the Fourier transform of a continuous-time signal $f(t)$.

[2.0 Marks]

b) Determine the Laplace transform of $f(t) = u(t) - u(t-T)$ where $u(t)$ denotes the unit step function.

[3.0 Marks]

c) Determine the current $i(t)$ in the circuit shown in Figure Q4 using the Laplace transform. Assume that $V_0(t) = u(t) - u(t-T)$ and the initial current in the circuit $i(0^-) = 0$

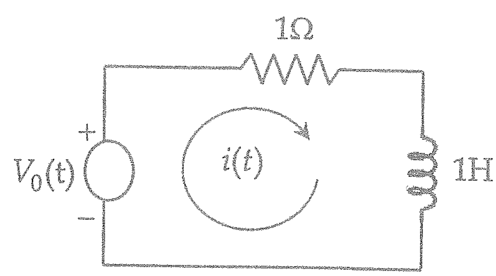

Figure Q4

[5.0 Marks]

Q5 a) Discuss the advantages of digital signal processing over analog signal processing of a continuous-time LTI system using block diagrams of the two processes.

[3.0 Marks]

b) i) "For sampling a continuous-time signal, the Nyquist sampling theorem is derived considering ideal sampling of the continuous-time signal." What is meant by the term "ideal sampling" in the aforesaid sentence?

ii) Explain briefly the two sampling techniques "natural sampling" and "flat-top sampling" used in practice.

[4.0 Marks]

c) Fourier spectra of two continuous-time signals $g_1(t)$ and $g_2(t)$ are shown in Figure Q5. Determine the Nyquist interval and sampling rate for the following signals.

i) $g_1(t)$

ii) $g_2(t)$

iii) $g_1(t)g_2(t)$

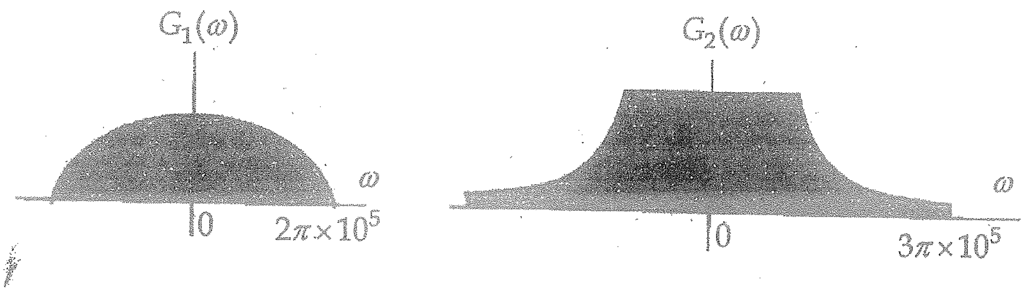

Figure Q5

[3.0 Marks]