University of Ruhuna

B.Sc. Degree in Fisheries and Marine Sciences
Level I(Semester II) - Examination- December 2016
(for repeat students)

Subject: Mathematics for Biology
Course Unit: FSC1b50 **Time:Two (02) Hours**

Answer four (04) questions only. Each question carries equal marks.
(Calculators will be provided)

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1. a) Consider the following set of numbers.

$$A = \{-7, 43, -\frac{4}{9}, \sqrt{2}, 1.33\ldots, 8^{1/3}, 0.75, \sqrt{-4}\}.$$

Simplify the elements of A as necessary and determine the following subsets.

(i) Natural numbers,
(ii) Integers,
(iii) Rational numbers which are not integers,
(iv) Irrational numbers.

b) Using the binomial theorem, find the coefficient of x^3 of the expansion of $\left(2x^3 + \frac{1}{x}\right)^5$.

c) (i) Express $\frac{1}{2}\log_a x + 4\log_a y - 3\log_a x$ as a single term of logarithm and simplify.

(ii) Solve the following equations.

i. $\log_3(2x-1) - \log_3(x-4) = 2$,

ii. $3^{x^2+4x} = \frac{1}{27}$

d) Find the real and imaginary parts of the complex number

$$\frac{1+2i}{3-4i} + \frac{2-i}{5i}.$$

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2. a) Constructing appropriate triangles show that

(i) $\sin 45^0 = \cos 45^0 = \frac{1}{\sqrt{2}}$,
(ii) $\sin 30^0 = \frac{1}{2}$, and
(iii) $\cos 30^0 = \frac{\sqrt{3}}{2}$.

b) (i) Using the trigonometric identities $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ with suitable values for α and β show that

i. $\sin 15^0 = \frac{\sqrt{3}-1}{2\sqrt{2}}$,

ii. $\sin 2x = 2\sin x\cos x$.

(ii) Show that $(\sin 15^0 + \cos 15^0)^2 = \frac{3}{2}$.

(iii) Show that $\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{2}{\sin 2x}$.

c) Determine the domain of each of the following real-valued functions:

(i) $f(x) = \sqrt{x-5}$

(ii) $f(x) = \sqrt{-x} + \frac{2}{x+1}$

Continued.

3. a) Find the following limits:

(i) $\lim_{x\to 2}\dfrac{x^2-2x}{x^2-x-2}$

(ii) $\lim_{x\to 5}\dfrac{\sqrt{x-1}-2}{x-5}$

(iii) $\lim_{x\to\infty}\dfrac{4-3x}{7+x}$

b) The derivative of a function $y=f(x)$ with respect to x is given, in the usual notation, by

$$\frac{dy}{dx}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}.$$

Using this formula show that, if $y=ax^2+bx+c$, then $\dfrac{dy}{dx}=2ax+b$, where a, b and c are constants.

c) Find the gradient (slope) of the tangent line to the graph of $y=x^3-4x$ at $x=2$. Show that the equation of the line is given by $y=8x-16$.

d) Differentiate the following functions:

(i) $f(x)=x^2+\frac{1}{x}$

(ii) $f(x)=(\sqrt{x}+4)(5x^2+x)$

(iii) $f(x)=\dfrac{(x-4)^2}{5x+2}$

(You need not to simplify the answers).

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4. a) The number of Salmon swimming upstream to spawn is approximated by the function

$$S(x)=-x^3+3x^2+360x+5000, \qquad (6\le x\le 20);$$

where x represents the temperature of water in degrees Celsius.

(i) Taking $y=S(x)$ with the above function, show that the critical points (turning points) of the curve are $x=-10$ and $x=12$.

(ii) Using the signs of the derivatives of corresponding regions or using the second derivative test classify the critical points as maxima or minima.

~~(iii) Sketch the graph of $y=S(x)$ for the region $-\infty<x<\infty$.~~

(iv) Giving reasons, determine the temperature that produces the maximum number of Salmon swimming upstream.

b) Consider the function given by

$$f(x,y)=(x^2+4y^2)^{1/3}.$$

(i) Find the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

(ii) Show that the total differential of f at the point $(2,-1)$ is given by

$$df=\frac{1}{3}(dx-2dy).$$

Continued.

c) (i) Evaluate the indefinite integral $\int 5x^{2/3}\, dx$.

(ii) Using the substitution $u = 2x^2 + 1$ evaluate the integral $\int \frac{4x}{\sqrt{2x^2+1}}\, dx$.

(iii) Using the method of integration by parts evaluate the definite integral $\int_0^1 (2x+1)e^x\, dx$.

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5. a) Consider the differential equation

$$\frac{2y}{x}\frac{dy}{dx} = \sqrt{5 + x^2}.$$

(i) Using the technique of separation of variables, find the general solution.

(ii) If $y = 3$ when $x = 2$ find the particular solution of the differential equation.

b) Consider the data sample $x_1, x_2, \ldots, x_{20}$ given below.

41	44	11	20	47
35	39	16	26	12
19	13	42	38	13
18	49	17	14	26

Here $\sum_{i=1}^{20} x_i = 540$ and $\sum_{i=1}^{20} x_i^2 = 17962$.

Find the mean, median, modes, variance and the standard deviation of the sample.
