UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: July 2017

Module Number: CE 5204 **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries 12 marks]

Q1. a) What is a yield line pattern?

[1 Mark]

b) How do you establish a yield line pattern in a reinforced concrete slab?

[2 Marks]

c) An orthotropic reinforced concrete slab continuous over four adjacent edges. One of the possible yield line patterns at collapse of the slab is shown in Figure Q1(a). The yield moments per unit length of reinforcements, which are provided to resist sagging moment and hogging moment of the slab, are m and m', respectively, as shown in Figure Q1(a). *Assume that* $m'=2m$, *and* $\mu=0.5$

i) Determine β and the ultimate uniformly distributed load that can be carried by the slab.

ii) If the slab continuous over three adjacent edges and the remaining edge is simply supported, a possible yield line pattern is identified as shown in Figure Q1(b). Determine α and γ and establish the exact yield line pattern.

iii) What is the collapse load corresponding to the yield line pattern shown in Figure Q1(b).

[9 Marks]

Q2. a) What are the merits of plate element over beam element with respect to the load resistance mechanisms?

[2 Marks]

b) A thin rectangular plate having a size of a x 2a is simply supported along all four edges. The plate carries a vertically downward load of intensity $q(x, y)$, which varies in the X and Y directions as given by

$$q(x,y) = q_0 Sin(\frac{\pi x}{a})Sin(\frac{\pi y}{2a})$$

where q_0 is the intensity of the load at the centre of the plate.

i) Assume a trail solution for displacement and show that the trail solution satisfies the relevant displacement and boundary conditions.

ii) Determine deflection of the plate. Hence, determine bending moments and shear forces.

iii) What is the total vertical reaction at the supports? Compare the total vertical reaction with the total applied load.

Governing equation and the equations for bending moments and shear forces (with usual notations and sign convention) are given by

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q}{D} \qquad M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$Q_x=-D\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x \partial y^2}\right) \qquad Q_y=-D\left(\frac{\partial^3 w}{\partial x^2 \partial y}+\frac{\partial^3 w}{\partial y^3}\right)$$

where ,

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[10 Marks]

Q3. a) What are the applications of a thin circular plate?

[2 Marks]

b) A circular plate of diameter d with a concentric hole of diameter d_h is fixed along the inner boundary and free along the outer boundary. The plate, manufactured with a material having poison ratio of 0.2, is used to be resisted a uniformly distributed vertically downward load of q per unit area. For a value $\frac{d}{d_h}=3$,

i) Determine the bending moment at the fixed edge, by considering a radial strip as a beam with the loading and end connection as in the plate.

ii) Determine the exact radial moment at the fixed edge.

iii) Discuss suitability of the solution determined in Part b(i) and Part b(ii) for the design purpose.

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{Q}{D} \quad M_r=-D\left(\frac{d^2 w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right) \quad M_t=-D\left(\frac{1}{r}\frac{dw}{dr}+\nu\frac{d^2 w}{dr^2}\right)$$

where

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[10 Marks]

Q4. a) Discuss the function of ring beams in a shell structure.

[2 Marks]

b) Determine the stress resultants in the spherical dome structure shown in Figure Q4.

Assume that the membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_\phi}{r_1}+\frac{N_\theta}{r_2}=P_r \qquad P_\phi r r_1 - r_1 N_\theta Cos\phi+\frac{\partial(rN\phi)}{\partial\phi}=0$$

[8 Marks]

c) Compute the forces developed at the horizontal ring beams.

[2 Marks]

Q5. a) Discuss with examples the classification of shell surfaces.

[2 Marks]

b) Show that the membrane stresses in a conical shell (*with usual notations and sign convention*) are given by

$$N_\theta = P_r\, S \tan\alpha$$

$$N_S = \frac{1}{S}\int (P_r\, S \tan\alpha - P_s\, S)\,ds$$

[4 Marks]

c) A conical shell made of thin aluminum sheets is proposed to be used as a roof of a cafeteria as shown in Figure Q5. The shell is subjected to its self-weight of "w" per unit surface area, and possible imposed load of "0.5 w" per unit plan area.

From the membrane theory derived in Part (b), determine membrane stress resultant in the roof shell structure.

[6 Marks]

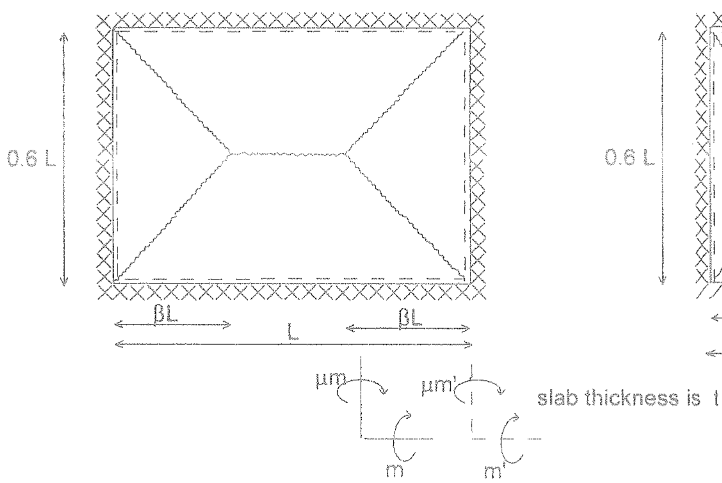

Figure Q1 (a)

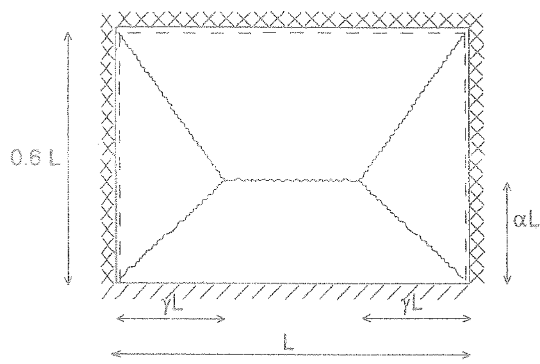

Figure Q1 (b)

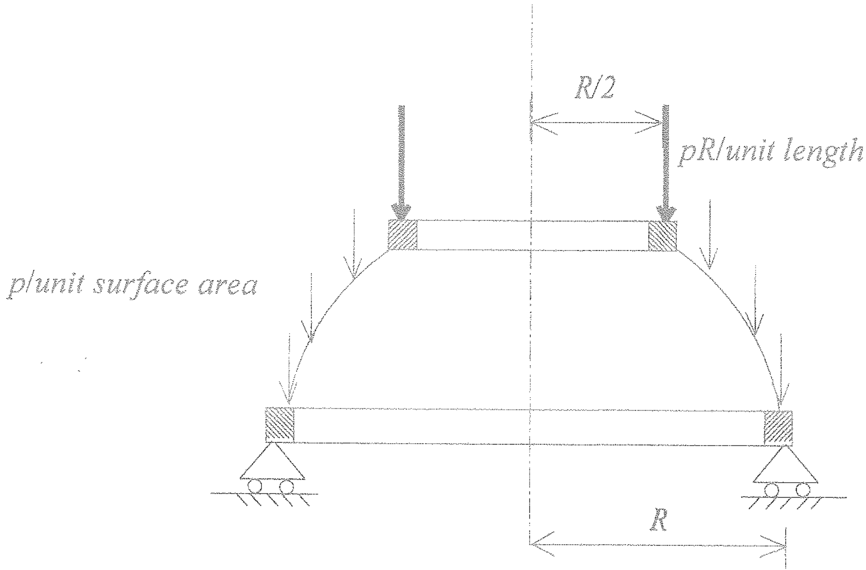

Figure Q4

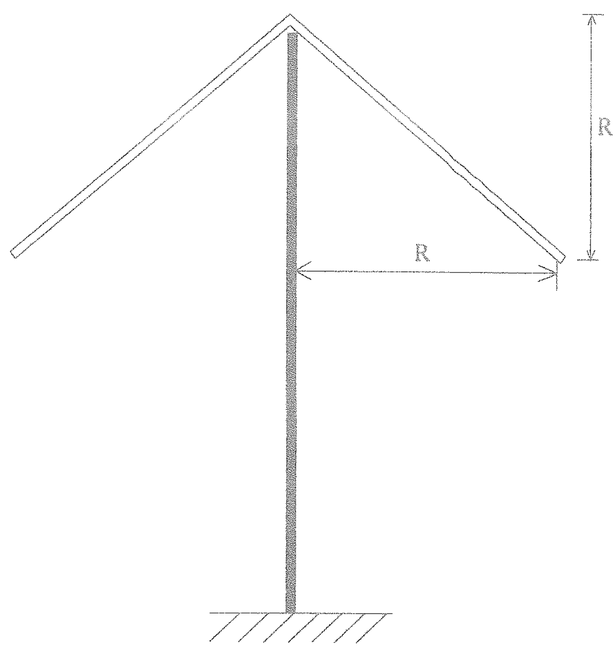

Figure Q5