UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 6 Examination in Engineering: November 2017

Module Number: ME 6302 **Module Name: Automatic Control Engineering**

[Three Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. You may make additional assumptions, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;
$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);
Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

A partial Laplace transform table is attached to this question paper.

Q1. a) The Figure Q1 shows a mechanical system having a rotational spring, a mass, and a rotational damper. The rotational damping coefficient of the damper is 'B' and rotational spring constant of the spring is 'K'. The mass has inertia of 'J'. The mass of the system was given an external torque of T Nm.

i. Obtain the transfer function of the system and transform it into Laplace domain.

[2.0 Marks]

ii. State the assumptions you made during (i) above.

[1.0 Marks]

iii. Obtain the expressions for ω_n and ζ in terms of K, B, and J.

[2.0 Marks]

iv. If K=6 Nm/rad and the system performance shows 33% overshoot and 2s settling time under unit step external torque input, find the values of J and B.

[2.0 Marks]

b) A DC motor connected to a mobile robot has the transfer function in the form of $G(s) = \frac{\omega(S)}{V(S)} = \frac{k_m}{\tau_m S+1}$ where $\omega(S)$, $V(S)$, k_m and τ_m stands for shaft speed, supply voltage, motor constant and time constant, respectively. When the supply voltage v(t) of the motor is kept at 5 V, the steady state speed of the motor is 80 rad/s. When the motor is started (i.e. Switched ON at t=0 s with a step input of 5 V) at 0 rad/s of shaft speed; the motor takes 3 seconds to reach 40 rad/s speed. Find the values of k_m and τ_m.

[5.0 Marks]

Q2. a) Dynamics of an open loop system is given by $K/_{S(S+5)}$.

i. Obtain the characteristic equation of closed loop unity negative feedback system.

[1.0 Marks]

ii. Find the break-away point and the corresponding value of the K from the characteristic equation obtained in (i) above.

[2.0 Marks]

iii. Find the roots of the characteristic quotation when K has values 0, 5, 10, and 15.

[2.0 Marks]

iv. Draw the root locus of the system (You can use your answer book to draw the root locus indicating important points in S-plane (Graph sheets are not necessary)

[3.0 Marks]

b) Figure Q2 shows pole locations P1 to P8 in S-plane. Draw the time domain response due to each individual pole comparing to each other and comment on the system stability, the frequency of oscillation, and settling time in each case compared to other points.

[4.0 Marks]

Q3. a) A system with a single feedback gain has open loop transfer function $G(S) = \frac{1}{(S+3)(S+4)}$.

i. Draw the closed loop negative feedback system block diagram with a single feedback gain and obtain the characteristic equation.

[1.0 Marks]

ii. If 6 percent overshoot and 2 percent settling time under 2 s are expected from the plant, obtain the required roots of the characteristic equation.

[3.0 Marks]

iii. Do the roots obtained above lie on the root locus of the characteristic equation obtained in (i) above? Justify your answer.

[1.0 Marks]

iv. To obtain the stated performance in (ii) from the system, a compensator has been used. Draw the closed loop system block diagram including the compensator. Clearly show the generic transfer function of the compensator in the block diagram.

[1.0 Marks]

v. Design a compensator to meet the required system performance as stated in (ii) above. The compensator requires a zero located at $(-5, j0)$.

[4.0 Marks]

b) Briefly describe the notch compensator (When/where and why it is used, how it operates and what is the information taken from a real system to design a notch compensator).

[2.0 Marks]

Q4. a) A plant has the following transfer function.

$$G(s) = \frac{2}{S(S^2 + 3S + 7)}$$

The closed loop system has a proportional controller $C(S)$ having a single proportional gain K and uses unity negative feedback.

i. Draw the system block diagram indicating controller and system dynamics and obtain the open loop transfer function.

[1.0 Marks]

ii. Obtain phase and magnitude of the frequency response of the open loop system.

[2.0 Marks]

iii. Find the value of the gain K which will make system marginally unstable.

[3.0 Marks]

iv. Draw the polar plot of the system as ω is varied from zero to infinity. A graph paper is attached to the end of this question paper (Use several check points to identify path of the polar plot).

[2.0 Marks]

v. Explain the gain margin, phase margin, and stability using the polar plot constructed in (iv) above.

[1.0 Marks]

b) Explain the advantages of frequency response analysis with reference to real-world control systems design.

[3.0 Marks]

Q5. Figure Q5 shows a one (1) degree of freedom magnetic levitation system constructed using a permanent magnet (PM). The system keeps the ball levitated in the air by rapidly moving the PM up and down using a voice coil motor (VCM). A VCM is a linear motor and has a mover which will move linearly. The force acting on the mover of the VCM (F_v) depends on the supplied current and the mover position (i.e. $F_v = f(i, z)$). The force acting on the steel ball depends on the air gap δz and can be expressed as $F_m = f(\delta z)$. A spring having spring constant of K is connected to the mover of VCM as shown. The link between the PM and mover of VCM is a rigid link.

a) Assuming the magnetic force acting on the ball can be represented as

$$F_m = f(\delta z) = {}^{a}/_{(b + c.\delta z^2)}$$

where a, b and c are constants.

Explain how to obtain the numerical values of a, b and c constants. Specify software tools if any and critical configuration details which will affect the accuracy and speed of model identification.

[2.0 Marks]

b) Explain how you would build a non-linear model for simulations using a block diagram. In your answer include details about software modules, critical configuration details, locations of sensors, joints and forces. State the expected function of each block.

[4.0 Marks]

c) Indicating system modeled in (b) using a single block, draw the control system block diagram using suitable controllers to achieve zero power levitation. Assume ideal conditions for all signals and clearly specify each signal.

[2.0 Marks]

d) The motor driver of VCM has a current limit of ± 5 A and has a transfer function specified by the manufacturer. Draw the block diagram of the system including blocks for noise simulation, filtering, and motor driver.

[2.0 Marks]

e) Briefly explain how you would tune the controller of system modeled in (d) to achieve stable zero power levitation under the noisy feedback.

[2.0 Marks]

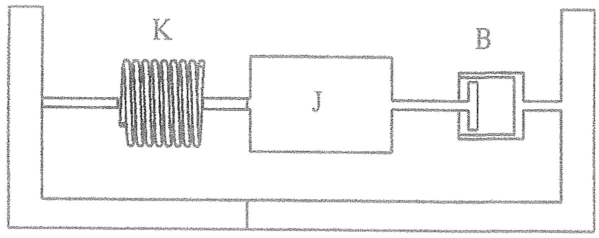


Figure Q1

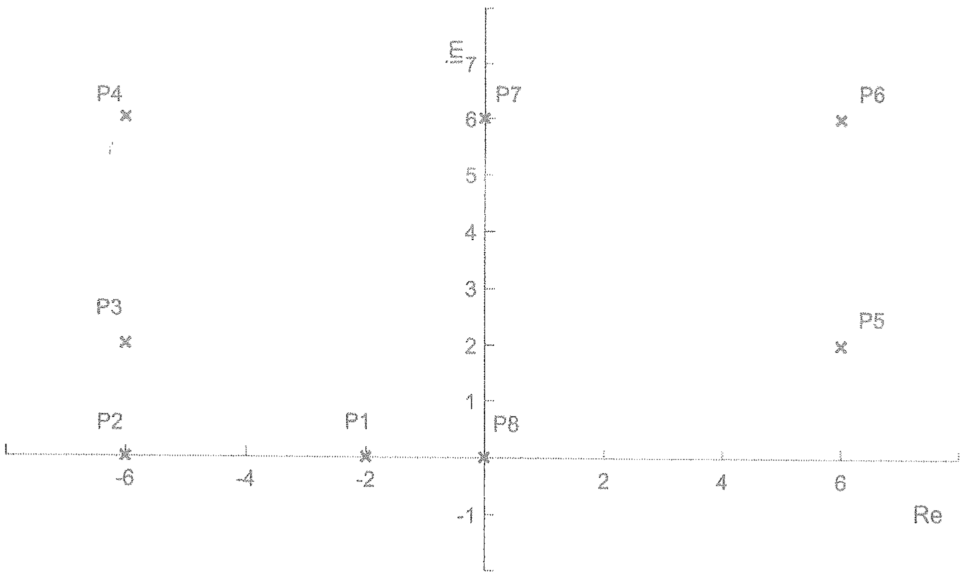


Figure Q2

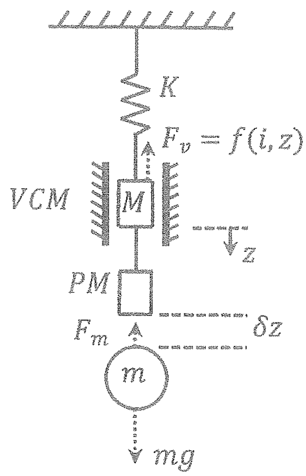
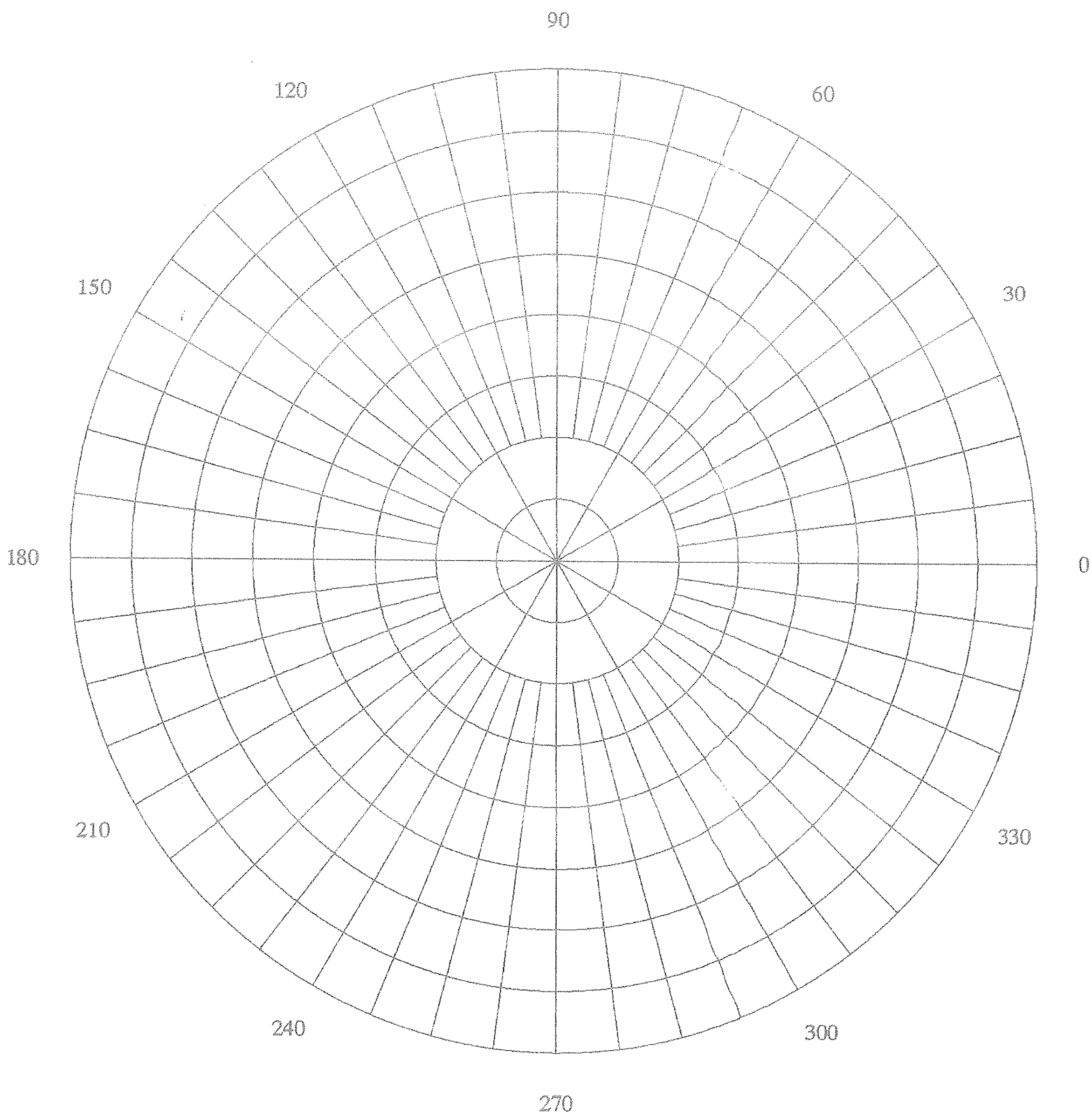


Figure Q5


90
120
60
150
30
180
0
210
330
240
300
270

Laplace transforms – Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		