UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2021

Module Number: ME 3305 **Module Name: Modelling and Controlling of Dynamic Systems**

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:

Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1. a) Figure Q1 shows a circuit with a resistor, a capacitor and an inductor. The voltage across a capacitor is described by $v_c(t)$ and the input voltage from the battery is $v(t)$. Current in network is $i(t)$.

i. Describe the $v_c(t)$ in terms of charge $q(t)$ of the capacitor and other suitable parameters.

[1 Mark]

ii. Describe the $v_c(t)$ in terms of current through the capacitor and other suitable parameters.

[1 Mark]

iii. Develop a mathematical model describing behavior of the circuit relating input voltage from the battery and output voltage across the capacitor in time domain.

[2 Marks]

iv. Obtain the transfer function of the system in Laplace domain

[2 Marks]

v. State any assumptions you made to obtain answer for (iv) above.

[1 Mark]

b) A system is described by $\frac{dc(t)}{dt} + 2c(t) = r(t)$ where $c(t)$ is output and $r(t)$ is input.

i. Obtain the transfer function of the system in Laplace domain and state any assumptions.

[1 Mark]

ii. If this system was excited by a unit step, obtain the output of the system in Laplace domain.

[1 Mark]

iii. Obtain the time domain response of the system using (ii) above.

[2 Marks]

iv. Plot the time domain response indicating important points. (Graph sheets are not necessary. Use free hand sketch.)

[1 Mark]

Q2. a) The rotational mass, damper and spring system is shown in Figure Q2. The mass of the system with the polar moment of inertia J was excited by an external torque of $T(t)$ Nm. The rotational spring constant and the damping constant are given by K and B respectively.

i. Obtain the transfer function of the system and transform it into Laplace domain assuming output of the system is angular position of the mass $\theta(t)$ and input is the torque $T(t)$.

[2 Marks]

ii. Obtain the natural frequency and damping ratio of the system.

[2 Marks]

iii. If this system must have settling time of 3 seconds and maximum of 30% overshoot, under a unit step torque input, find the values of polar moment of inertia and damping constant of the system when the rotational spring constant is 3 Nm/rad.

[3 Marks]

iv. Obtain the poles and zeros of the system and state them separately.

[2 Marks]

b) Transfer function of a system is given by $G(s) = \frac{a}{s+a}$. The system was excited by a unit step input at time $t = 0$. Obtain the response of the system at the time $t = {}^{1}/_{a}$.

[3 Marks]

Q3. a) The mass-damper system shown in the figure Q3-Q4 is subjected to the forcing function $\hat{f}(t) = \hat{F}.\cos(\omega t)$. The displacement of mass is $\hat{x}(t)$.

i. State the forcing function in complex exponential form.

[1 Mark]

ii. Obtain the mathematical model describing motion of the mass in time domain.

[1 Mark]

iii. Explain why it is possible to use $\hat{x}(t) = Ae^{j\omega t}$ with same ω as $\hat{f}(t)$ to replace $\hat{x}(t)$ in (ii) above when the system is in steady state.

[1 Mark]

iv. Obtain an expression to describe complex displacement $\hat{x}(t)$.

[3 Marks]

v. The mechanical **impedance** is defined as $\hat{z} = {}^{\hat{f}}/_{\hat{v}}$ where $\hat{v}$ is complex velocity. Obtain the mechanical **reactance**.

[3 Marks]

b) Harmonic oscillation of a system is described as superposition of following two models.

$$\hat{x}_1 = A_1 e^{j(\omega t+\varphi_1)} \qquad \hat{x}_2 = A_2 e^{j(\omega t+\varphi_2)}$$

i. Draw the phasor diagram to show superposition of two harmonics.

[1 Mark]

ii. Obtain the amplitude A and phase φ of the vibration of entire system.

[2 Marks]

Q4. The mass-damper system shown in figure Q3-Q4 is subjected to $\hat{f}(t) = u$

Notes:

1. Inverse of Matrix A is $A^{-1} = \frac{Adjoint\ (A)}{Determinant(A)}$
2. Transpose of cofactor matrix is defined as adjoint matrix.
3. C_{ij} th element of cofactor matrix is defined as $C_{ij} = (-1)^{(i+j)} \times M_{ij}$ where, M_{ij} is the respective Minor).
4. Minor M_{ij} of 2×2 matrix can be obtained by removing i th row and j th column from 2×2 matrix.

a. Draw the free body diagram, indicating forces acting on the mass.

[1 Mark]

b. Obtain the state space model of the system taking x and $\dot{x}$ as states.

[2 Marks]

c. Taking m = 2 kg, B = 8 Ns/m, K = 6 N/m, $\hat{f}(t) = u = 0$ obtain the characteristic equation using the state space model obtained above.

[2 Marks]

d. Obtain the natural frequency and damping ratio of the system comparing your answer to (c) with standard second order form.

[2 Marks]

e. Obtain the state transition matrixes $\emptyset(s)$ and $\emptyset(t)$.

[4 Marks]

f. Based on the results of (e), state whether this system is stable or not and give reasons for your answer.

[1 Mark]

Q5. a) A nonlinear system dynamics is given as $\dot{x} = F(x) = \cos(x)$.

i. Determine the stability of the system around operating point $x_1 = {}^{\pi}/_{2}$ rad.

[2 Marks]

ii. Determine the stability of the system around operating point $x_2 = {}^{3\pi}/_{2}$ rad.

[2 Marks]

iii. Draw the graph showing x vs $F(x)$ and use the results above to indicate the direction of motion of system when x is between -2π rad to $+2\pi$ rad using arrow heads on the x axis.

[2 Marks]

b) A dynamic system is given by $\dot{x} = F(x) = rx(1 - x)$ where r is a positive real valued constant.

i. Find the fixed points (points where velocity is zero) of the system.

[1 Mark]

ii. Find the Jacobian function of the system.

[1 Mark]

iii. Find the eigenvalues of the Jacobian function.

[2 Mark]

iv. State the stability of each fixed point with reasons.

[2 marks]

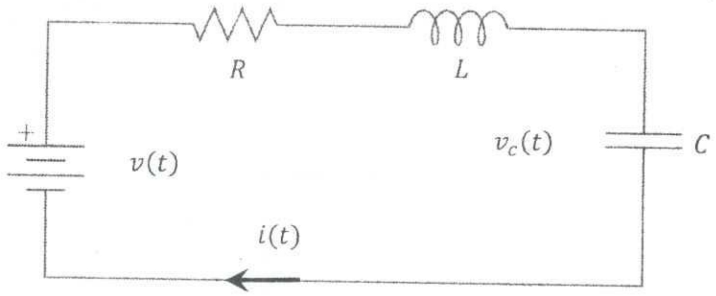

Figure Q1

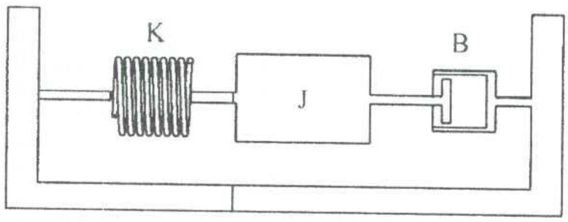

Figure Q2

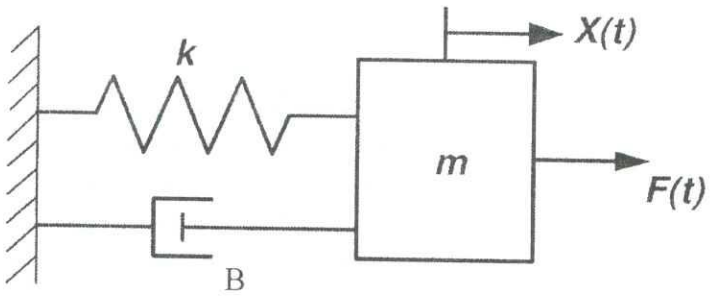

Figure Q3 - Q4

Laplace transforms – Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 - \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 - \omega^2}$
e^{-at}	$\frac{1}{s - a}$	$\sin(\omega t - \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 - \omega^2}$
te^{-at}	$\frac{1}{(s-a)^2}$	$\cos(\omega t - \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 - \omega^2}$
$\frac{1}{2}t^2e^{-at}$	$\frac{1}{(s-a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 - \omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s-a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 - \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s-a)(s-b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 - at)]$	$\frac{1}{s(s-a)^2}$	$e^{-at} \cos \omega t$	$\frac{s-a}{(s-a)^2 - \omega^2}$
t^n	$\frac{n!}{s^{n-1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n-1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 - \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s-a)^{n-1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s-a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 - e^{-at})$	$\frac{1}{s^2(s-a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		