UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2021

Module Number: EE3305 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions**, each question carries 10 marks]

Q1 a) i) State a mathematical expression to determine the energy of a continuous time signal.

[2 Marks]

ii) An arbitrary real valued continuous time signal is represented by

$$f(t) = f_e(t) + f_o(t)$$

where $f_e(t)$ and $f_o(t)$ are the even and odd components of $f(t)$. The signal $f(t)$ occupies the entire interval $-\infty < t < \infty$. Show that the energy of the signal $f(t)$ is equal to the sum of the energy of the even component $f_e(t)$ and the energy of the odd component $f_o(t)$.

[2 Marks]

b) Consider the interconnection of an LTI system shown in Fig. Q1.

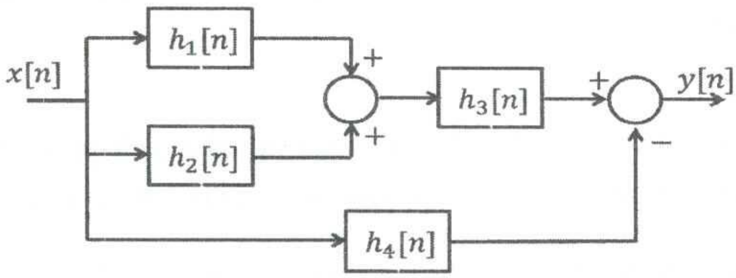

Fig. Q1

i) Determine the impulse response of the overall system in terms of $h_1[n]$, $h_2[n]$, $h_3[n]$ and $h_4[n]$.

[3 Marks]

ii) If $h_1[n] = u[n]$, $h_2[n] = u[n+2] - u[n]$, $h_3[n] = \delta[n-2]$ and $h_4[n] = \alpha^n u[n]$, determine the impulse response of the overall system.

[3 Marks]

Q2 a) Fourier Series (FS), Discrete Time Fourier Series (DTFS), Fourier Transform (FT) and Discrete Time Fourier Transform (DTFT) are the four distinct Fourier representations for different classes of signals. State the appropriate Fourier representations (i.e. FS, DTFS, FT or DTFT) for the following signals with justifications.

i) $f(t) = 1 - \cos(2\pi t) + \sin(3\pi t)$

[2 Marks]

ii) $f[n] = \left(\frac{1}{2}\right)^n u[n]$

[2 Marks]

b) Consider the LTI system shown in Fig. Q2.

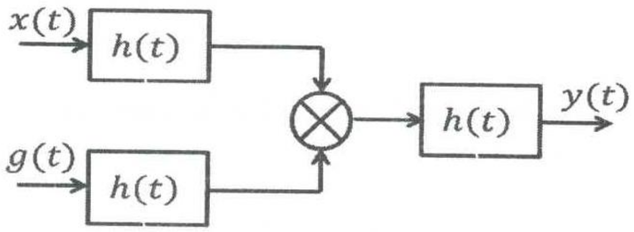

Fig. Q2

The impulse response of each sub system is $h(t) = \frac{\sin(11\pi t)}{\pi t}$ and the two inputs to the system are $x(t) = \sum_{k=1}^{\infty} \frac{1}{k^2} \cos(k5\pi t)$ and $g(t) = \sum_{k=1}^{10} \cos(k8\pi t)$.

i) Determine the Fourier transforms of $h(t)$, $x(t)$ and $g(t)$. Refer the Fourier transform pairs shown in Table 1.

[3 Marks]

ii) Using the results obtained in part b) i), determine the overall system output $y(t)$.

[3 Marks]

Q3 a) The Unilateral Laplace transform of a signal $f(t)$ is given by $F(s)$.

i) Show that the Laplace transform of $f(t)e^{-at}$ is $F(s-a)$.

[1 Mark]

ii) Determine the Laplace transform of $z(t) = e^{-at}u(t)$. Show all mathematical derivations in your answer.

[2 Marks]

iii) Using the results obtained in part a) ii), determine the Unilateral Laplace transform of $y(t) = e^{-at}\cos(\omega_0 t)u(t)$.

[2 Marks]

b) Consider the circuit diagram shown in Fig. Q3. Assume that the initial conditions are $i_L(0) = 1\text{ A}$ and $v_c(0) = 3\text{ V}$.

i) Draw the transformed circuit in s-domain by representing all the voltages and currents by their Laplace transforms.

[2.5 Marks]

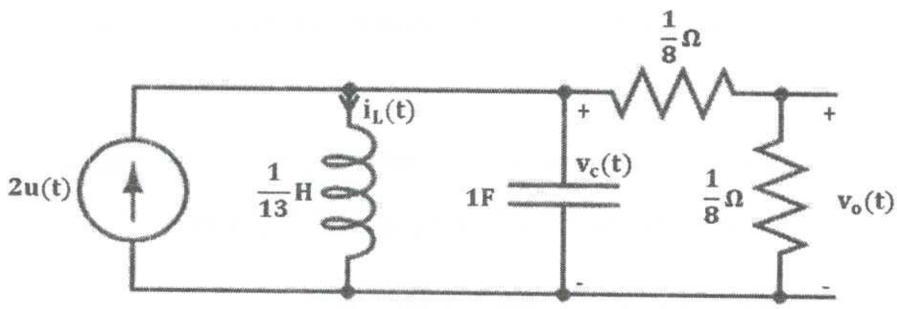

Fig. Q3

ii) Determine the output voltage $V_o(s)$ in s-domain.

Hint: A capacitor C with an initial condition $v_c(0)$ can be represented by an uncharged capacitor of impedance $\frac{1}{Cs}$ in parallel with a current source $Cv_c(0)$. Similarly, an inductor L with an initial current $i_L(0)$ can be represented by an inductor of impedance Ls in parallel with current source $\frac{i_L(0)}{s}$.

[2.5 Marks]

Q4 a) i) Briefly explain why discrete-time signal processing is required for continuous time signals.

[2 Marks]

ii) Explain why low pass filtering is needed before obtaining samples of a continuous time signal.

[2 Marks]

b) Suppose a signal $f(t)$ is uniquely represented by a discrete sequence

$$f[n] = f(nT_s)$$

where T_s is the sampling interval. Determine the conditions to be satisfied on the T_s for the following signals.

i) $f(t) = \frac{\sin(10\pi t)}{\pi t}$

[3 Marks]

ii) $f(t) = \cos(\pi t) + 3sin(2\pi t) + \sin(4\pi t)$

[3 Marks]

Q5 a) i) State a mathematical expression for the z-transform of a discrete time signal $f[n]$.

[1 Mark]

ii) Determine the z-transform of $\left(\frac{1}{3}\right)^n u[n]$.

[2 Marks]

iii) Sketch the Region of Convergence (ROC) for the part a) ii).

[2 Marks]

b) The Discrete Time Fourier Transform (DTFT) of a signal $f[n]$ is given by $F(\Omega)$.

i) Determine the DTFT of the signal $\left(\frac{1}{2}\right)^n u[n]$.

[1 Mark]

ii) Show that $f[n]\, e^{jn\Omega_0} = F(\Omega - \Omega_0)$.

[2 Marks]

iii) Use the results in part b) i) and b) ii) to determine the DTFT of the signal $\left[\left(\frac{1}{2}\right)^n \cos\frac{\pi n}{2}\right] u[n]$.

<u>Hint:</u> The geometric progression for a common ratio α is $\sum_{n=0}^{\alpha} \alpha^n = \frac{1}{1-\alpha}$ for $|\alpha| < 1$.

[2 Marks]

Table 1: Common table of Fourier transforms

	$f(t)$	$F(\omega)$	
1	$e^{-at}u(t)$	$\dfrac{1}{a+j\omega}$	$a>0$
2	$e^{at}u(-t)$	$\dfrac{1}{a-j\omega}$	$a>0$
3	$e^{-a\|t\|}$	$\dfrac{2a}{a^2+\omega^2}$	$a>0$
4	$te^{-at}u(t)$	$\dfrac{1}{(a+j\omega)^2}$	$a>0$
5	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}}$	$a>0$
6	$\delta(t)$	1	
7	1	$2\pi\delta(\omega)$	
8	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
9	$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	
10	$\sin\omega_0 t$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$	
11	$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
12	$\mathrm{sgn}\, t$	$\frac{2}{j\omega}$	
13	$\cos\omega_0 t\, u(t)$	$\frac{\pi}{2}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]+\frac{j\omega}{\omega_0^2-\omega^2}$	
14	$\sin\omega_0 t\, u(t)$	$\frac{\pi}{2j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]+\frac{\omega_0}{\omega_0^2-\omega^2}$	
15	$e^{-at}\sin\omega_0 t\, u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a>0$
16	$e^{-at}\cos\omega_0 t\, u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a>0$
17	$\mathrm{rect}\left(\frac{t}{\tau}\right)$	$\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$	
18	$\frac{W}{\pi}\mathrm{sinc}(Wt)$	$\mathrm{rect}\left(\frac{\omega}{2W}\right)$	
19	$\Delta\left(\frac{t}{\tau}\right)$	$\frac{\tau}{2}\mathrm{sinc}^2\left(\frac{\omega\tau}{4}\right)$	
20	$\frac{W}{2\pi}\mathrm{sinc}^2\left(\frac{Wt}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
21	$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T}$
22	$e^{-t^2/2\sigma^2}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	