UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: December 2020

Module Number: ME 5301 **Module Name: Advanced Control Systems**

[**Three** Hours]

[Answer **all questions**, each question carries **twelve** marks]

Important:
Some necessary equations and a partial table of Laplace transformation pairs have been provided on the question paper. **You may make additional assumptions**, if necessary, by clearly stating them in your answers. Some standard notations may have been used without defining them.

The standard form of a second-order system is $G(s) = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$;

$T_s = \frac{4}{\zeta\omega_n}$ ($\pm 2\%$ settling time);

Percentage Overshoot = $e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100$;

Q1. a) Figure Q1 shows a circuit with two resistors and two capacitors in two connected networks. The voltage across a capacitor is described by $v(t) = \left(1/_C\right)\int i dt$. Current in network 1 is i_1 and current in network 2 is i_2.

i. Obtain the differential equation describing $v_3(t)$ in terms of parameters of network 2 and $v_2(t)$.

[1 Mark]

ii. Obtain the differential equation describing $i_1(t)$ in terms of values of resistors and capacitors of two networks and $v_2(t)$.

[2 Marks]

iii. Obtain the relationship between $v_1(t)$ and $v_2(t)$ in terms of component values of two networks.

[2 Marks]

iv. Obtain the Laplace transform of part (iii) above.

[1 Mark]

v. Obtain the unit step response of the network shown in Figure Q1 taking $v_1(t)$ as the input and $v_2(t)$ as the output.

[1 Mark]

vi. State any assumptions you made to obtain answers for parts (i) to (v) above.

[1 Mark]

b) A DC motor develops a torque $T(t)$ proportional to the supplied current $i(t)$. (i.e. $T(t) = K.i(t)$ where K is the motor torque constant.) The moment of inertia of the motor $J = 2.5\ kg.m^2$ and the damping coefficient $C = 0.5\ Nm.s/rad$. The rotational speed of the motor is ω.

i. Determine differential equation relating rotational speed ω and current i.

[1 Mark]

ii. When a constant 2 A current passes through the motor, the steady-state rotational speed of the motor is 7.5 rad/s. Obtain the value of torque constant K and developed torque T.

[3 Marks]

Q2. a) The dynamics of an open-loop system is given by the following transfer function. $G_P(s) = \frac{1}{2(S+4)}$. The system is driven by an amplifier having the transfer function $G_A(s) = \frac{1}{S}$.

i. Draw the block diagram of the respective closed-loop system having unity negative feedback and a single proportional controller having gain K in the forward path.

[1 Mark]

ii. Obtain the characteristic equation corresponding to the system obtained in (i) above.

[1 Mark]

iii. Obtain the break-away point of the root locus and the corresponding value of the gain K.

[2 Marks]

iv. Find the roots of the characteristic quotation when K has values 0, 4, 8, 12, and 16.

[2 Marks]

v. Draw the root locus of the system. (Use your answer book to draw the root locus indicating important points in the s-plane (Graph sheets are not necessary).

[3 Marks]

b) The characteristic equation of a system controlled by a single feedback gain K is shown below. Obtain the settling time of the system, when the controller gain K is selected such that both roots are equal.

$$1 + \frac{KS(S+4)}{S^2 + 2S + 2} = 0$$

[3 Marks]

Q3. a) Characteristic polynomial P of a system is given by $P(S) = S^4 + S^3 + S^2 + S + K$.

i. Obtain Routh's array of polynomial P(S).

[3 Marks]

ii. Determine the values of K which will make the system **unstable.**

[2 Marks]

b) A system G(S) is specified as $G(S) = K/(1 + TS)$

i. Obtain the frequency response of the system in $a + jb$ form.

[2 Marks]

ii. Obtain the magnitude and the phase of the system.

[2 Marks]

iii. Fill the following table using information from the above system G(S).

ω (rad/s)	$Magnitude$	$Phase$ (degrees)
0		
$1/_T$		
∞		

[1 Mark]

iv. Plot the frequency response of the system in a complex plane as a polar plot indicating important details.

[2 Marks]

Q4. a) The dynamics of an open-loop plant are modeled using the following transfer function.

$$G(s) = \frac{1}{(S+3)(S+9)}$$

This plant is controlled suing a proportional-only controller having gain K in the feedback path. Answer the following questions leading to the design of a suitable controller and a compensator to achieve specified performance from the plant.

i. Draw the closed-loop negative feedback system block diagram and obtain the open-loop transfer function of the system.

[1 Mark]

ii. Obtain the characteristic equation of the system.

[1 Mark]

iii. If overshoot under 9.5% and settling time under 0.8 s are expected from the system, obtain the required roots of the characteristic equation.

[3 Marks]

iv. Can the system satisfy the required performance of (iii) above by tuning feedback gain K? Prove your answer using graphically or mathematically.

[2 Marks]

v. A generic compensator of the form $\frac{S+10}{S+P}$ has been inserted before the system input, to compensate error signal. Draw the system block diagram and clearly show the location of the generic compensator.

[1 Mark]

vi. Design the compensator to meet the required system performance as stated in (iii) above. The compensator requires a zero located at $(-10, j0)$.

[3 Marks]

b) What type of controller/compensator required if the steady-state error of the response must be kept at zero?

[1 Mark]

Q5. Figure Q5 shows a 1 degree of freedom (DOF) ball balancing platform. The ball shown touches the inclined platform at location P. At the center of the platform, indicated by the R and arrowhead, the platform is pivoted to the pivot arm using 1 DOF rotary joint. The other end of the pivot arm is connected to the ground. The distance between P and R measured along the platform from R is x. Platform angle β can be changed by applying force $f(t)$. By measuring β and x, a controller can calculate the required force $f(t)$ to take the ball to the center of the platform, if the ball position was changed by an external disturbance force $f_d(t)$ acting on the ball (i.e. maintaining $x = 0$ at steady state). Friction is assumed to be zero during modelling and simulations.

a) Design a non-linear system dynamics model to simulate this ball balancing system by using a block diagram. In your answer, include details about critical configurations, locations of the sensors, joints, and forces. State the expected function of each of the blocks used. Inputs to the simulator are the Force $f(t)$ and external disturbance force $f_d(t)$. The outputs are x, and β.

[4 Marks]

b) Indicating the system modeled in (a) above by using a single block, draw a suitable closed-loop linear control system using a block diagram to keep the ball at the center of the platform while the platform is horizontal. i.e. your goal is to keep $x = 0$ and $\beta = 0$ at the steady state. (Hint: you can use two separate controllers to control angle β and distance x. Responses of linear controllers can be added together to achieve a single control signal $f(t)$)

[2 Marks]

c) Suggest suitable sensors to measure angle β and distance x with sufficient accuracy and precision. If you are asked to implement this system as a physical prototype, what is the expected measurement accuracy in each sensor?.

[3 Marks]

d) Due to measurement noise, distance x was measured using two sensors. You need to take the optimal angle possible by combining both sensor readings.

   i. What type of filter you will use to combine readings of both sensors and get the optimal result?

   [1 Mark]

   ii. Draw a block diagram of the filter you selected, having input x, output $\hat{x}$ and indicating important parameters, blocks, and signals.

   [2 Marks]

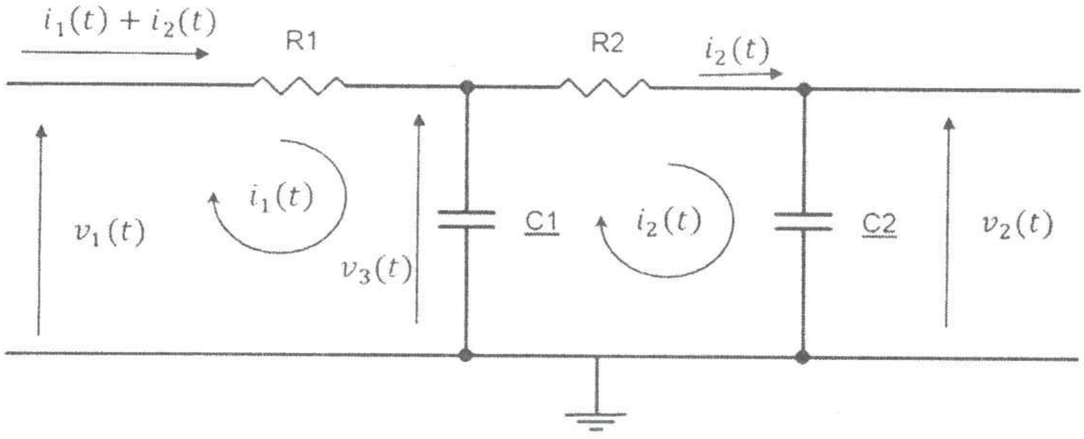

Figure Q1

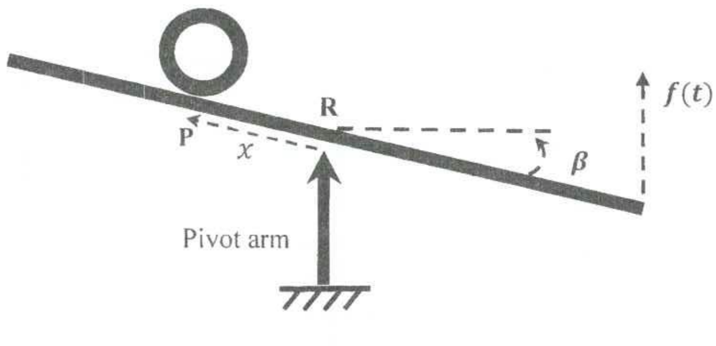

Figure Q5

Laplace transforms – Table

$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 - \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 - \omega^2}$
e^{-at}	$\frac{1}{s-a}$	$\sin(\omega t - \theta)$	$\frac{s \sin\theta - \omega \cos\theta}{s^2 - \omega^2}$
te^{-at}	$\frac{1}{(s-a)^2}$	$\cos(\omega t - \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 - \omega^2}$
$\frac{1}{2}t^2 e^{-at}$	$\frac{1}{(s-a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 - \omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s-a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 - \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s-a)(s-b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 - at)]$	$\frac{1}{s(s-a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s-a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n-1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 - \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n-1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s-a)^{n-1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s-a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 - e^{-at})$	$\frac{1}{s^2(s-a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) = f_2(t)$	$F_1(s) = F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} - \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2 f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^n f}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		