
A comparison of partition of unity finite element method and finite volume technique for solving 1-D heat conduction problems

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The scope of this research concerns with the numerical methods used for solving one-dimensional partial differential equations. It is motivated by the need for efficient numerical methods to deal with partial differential equations that are difficult to solve using analytical approaches. Among those numerical methods, the Finite Difference Method (FDM) is the simplest method, and the Finite Volume Method (FVM) is generally expected to provide better conservation properties. Also, the Partition of Unity Finite Element Method (PUFEM) has been identified as an extremely powerful new numerical method which deals with overlapping subdomains. In this study, we examined the PUFEM and FVM to see whether they produced equivalent numerical solutions. We considered two problems namely, a steady-state heat conduction problem and an unsteady state heat conduction problem when Dirichlet boundary conditions are given, for this study. In the Steady state heat transfer problem, we studied the change of the temperature of different points in the spatial domain, and in the unsteady state heat transfer problem, we examined the temperature distribution at a given point x in space and at a given time t . Both of these problems are solved separately using FDM, FVM and PUFEM. We found that the numerical solutions from the FDM are accurate by comparing the exact analytical solutions of one-dimensional steady state heat conduction problem with those from the FDM. The numerical solutions of FVM and FDM were compared to ensure that the FVM numerical solutions were accurate. Then the numerical solutions from PUFEM and FVM were compared, and we observed that the numerical solutions obtained using the two numerical approaches are indistinguishable in one dimension. We found that it is efficient to use PUFEM, which is a meshless method, instead of FVM, which is a mesh-based method, because PUFEM requires less computational time and programming effort.

Keywords: Partition of unity finite element method, Finite volume method, Finite difference method, Steady, Unsteady, Heat conduction

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