UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 8 Examination in Engineering: November 2017

Module Number: EE8210 **Module Name: Digital Communication**

[Three Hours]

[Answer **all questions,** each question carries **10 marks]**

Q1 a) Explain the importance of the orthogonality of signals in the detection process of a digital communication system.

[3.0 Marks]

b) i) Show that the three functions shown in Figure Q1 are pairwise orthogonal over the interval (-2,2).

ii) Determine the value of the constant A that makes the set of orthonormal functions.

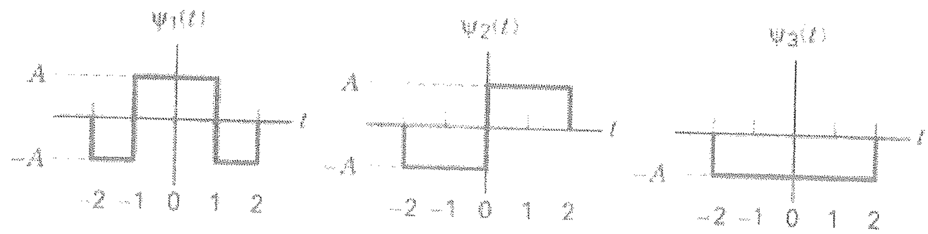


Figure Q1

iii) Express the waveform $x(t)$ in terms of the orthonormal set in part ii).

$$x(t) = \begin{cases} 1 & 0 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

[7.0 Marks]

Q2 a) When simulating a digital communication system, the following two functions are used.

$$Q(x) = P(X > x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{(-\omega^2/2)} d\omega$$

$$erf(x) \triangleq \frac{2}{\sqrt{\pi}} \int_0^x e^{(-t^2)} dt$$

Show that $Q(y) = \frac{1}{2} - \frac{1}{2} erf\left(\frac{y}{\sqrt{2}}\right)$ for $y = \sqrt{2}x$

[4.0 Marks]

b) Suppose that the receiver of a digital communication system uses the $X_2 = X_1 + N$ expression where $X_1 \in (0,1)$ with equal probabilities and N is independent of X_1. The mean and the variance of N are 1 and 0.5 respectively. Further, the receiver makes decisions based on the following criteria.

- If $X_2 \leq \frac{1}{3}$, then the receiver decides that the transmitter has sent a "0".
- If $X_2 > \frac{1}{3}$, then the receiver decides that the transmitter has sent a "1".

Determine the error probabilities at the receiver in terms of $Q(.)$ function for the following cases.

i) Given that $X_1 = 1$ the receiver decides the transmitter has sent a "0".

ii) Given that $X_1 = 0$ the receiver decides the transmitter has sent a "1".

[6.0 Marks]

Q3 a) A typical M-ary PAM (Pulse Amplitude Modulated) system uses the following amplitudes for its M signal points.

$$-(M-1)A,\ -(M-3)A,\ \dots\dots\dots,\ -A,\ A,\ \dots\dots\dots,\ (M-3)A,\ (M-1)A$$

The average symbol energy of the M-ary PAM system is given by

$$E_s = \frac{(M^2-1)}{3}A^2.$$

Determine the average bit energy (E_b) for the M-ary PAM system described by

$$\begin{aligned} s_0(t) &= 1.5[u(t) - u(t-1)] \\ s_1(t) &= 0.5[u(t) - u(t-1)] \\ s_2(t) &= -0.5[u(t) - u(t-1)] \\ s_3(t) &= -1.5[u(t) - u(t-1)] \end{aligned}$$

[5.0 Marks]

b) Identify and draw the decision regions for the signal constellation diagram shown in Figure Q3 by using pairwise comparison of binary detection for PAM signals.

[5.0 Marks]

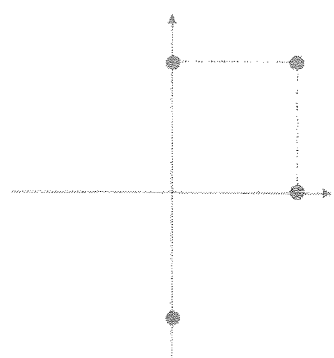


Figure Q3

Q4 a) "Determining the exact probability of error analysis for *M*-ary PSK (Phase Shift Keying) and QAM (Quadrature Amplitude Modulation) constellations are not straightforward and thus uses some approximations."
Discuss the validity of the above argument.

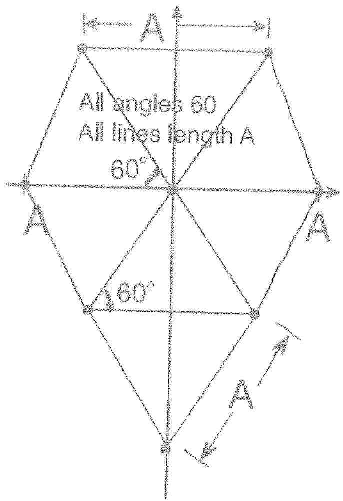
[3.0 Marks]

b) Determine the probability of symbol error for the signal space diagram shown in Figure Q4 using

i) Union bound approximation.

ii) Nearest-neighbour approximation.

[7.0 Marks]



FigureQ4

Q5 a) In M-ary FSK (Frequency Shift Keying), symbols are sinusoids with their frequencies selected among a set of M different frequencies $\{f_0, f_1, \ldots\ldots, f_{M-1}\}$. Therefore, consider a cosine signal with a frequency modulation of $f_k = f_c + k\Delta f$ and $Q_k(t) = \sqrt{2}\, p(t)\cos[2\pi f_c t + 2\pi k\Delta f t]$ where

$$p(t) = \begin{cases} \dfrac{1}{\sqrt{T_s}} & 0 \le t \le T_s \\ 0 & \text{otherwise} \end{cases}.$$

Determine Δf such that two FSK symbols are orthogonal to each other.

[4.0 Marks]

b) Explain how FSK signals are generated for transmission.

[2.0 Marks]

c) Explain the difference between the coherent reception and the non-coherent reception of FSK signal detection.

[4.0 Marks]