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Investigation of efficiency of the higher-level multigrid methods for solving a large system of linear equations which arises from discretizing numerical schemes of 1D Poisson equation

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Multigrid Methods (MG) are extremely effective numerical techniques in solving a large system of linear equations associated with boundary value problems in various fields such as engineering, physics, and medicine etc. The references show that, in general, the order of computational complexity of multigrid methods is $O(N)$ whereas classical linear system solver like Gauss-Seidel (GS) takes the order $O(N^2)$. The objective of this research is to implement the multigrid algorithm using MATLAB software and to solve a large linear system of equations using the implemented algorithm to evaluate the convergence nature of MG. In this study, a detailed investigation is carried out to show the efficiency of MG considering different levels of MG algorithm for 1D Poisson equation with Dirichlet boundary conditions. First, the problem was solved using classical iterative methods. The error behaviour and the required number of iterations in each case showed that such solvers are not good to solve associated linear system. Among the classical iterative solvers Gauss-Seidel is one of the fastest and the easiest solvers so it is chosen as the smoother for the multigrid methods. Then the two-level, three-level, and four-level v-cycle MG with GS as the smoother were implemented using MATLAB software. Secondly, the problem was solved using MG. Later, the solutions were obtained using the v-cycle multigrid methods. The obtained results indicate that the multigrid methods accelerate the convergence of the solution drastically.

Keywords: Multigrid, V-cycle, Iterative methods, Poisson equation, Dirichlet boundary conditions

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