UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 5 Examination in Engineering: July 2016

Module Number: CE 5204 **Module Name: Structural Analysis III**

[Three Hours]

[Answer **all questions**, each question carries 12 marks]

Q1. a) What is a positive yield line?

[1 Mark]

b) Explain how you identify the positive yield line in a reinforced concrete slab?

[1 Mark]

c) You are assigned a task to assess an existing reinforced concrete slab. The slab is orthotropically reinforced and supported as shown in Figure Q1. The yield moments per unit length of reinforcements, which are provided to resist sagging moment and hogging moment of the slab, are m and m' (=1.2 m), respectively. Yield moments for each direction of the slab are shown in Figure Q1, and parameter μ can be assumed as 0.5. Yield moment for beam reinforcements is assumed to be $1.5mL$.

i) Draw a possible yield line pattern for the slab shown in Figure Q1.

ii) Determine the exact distances between the supports and the intersecting points of the yield lines.

iii) Determine the ultimate uniformly distributed load that can be carried by the slab.

[10 Marks]

Q2. a) What is a "thin" plate?

[2 Marks]

b) A thin rectangular plate having a size of a x b is simply supported along all four edges. The plate carries a vertically downward load of intensity $q(x,y)$, which varies in the x and y directions as given by

$$q(x,y) = q_0 Sin(\frac{m\pi x}{a})Sin(\frac{n\pi y}{b})$$

where q_0 is the intensity of the load at the centre of the plate.

For m=n=1,

i) Assume a trail solution for displacement and show that the trail solution satisfies the relevant displacement and boundary conditions.

ii) Determine deflection of the plate.

iii) Determine bending moments and shear forces.

iv) What would be the deflection of the plate if m=1 and n=0?

Governing equation and the equations for bending moments and shear forces (with usual notations and sign convention) are given by

$$\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{q}{D} \qquad M_x=-D\left(\frac{\partial^2 w}{\partial x^2}+\nu\frac{\partial^2 w}{\partial y^2}\right) \qquad M_y=-D\left(\frac{\partial^2 w}{\partial y^2}+\nu\frac{\partial^2 w}{\partial x^2}\right)$$

$$Q_x=-D\left(\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 w}{\partial x \partial y^2}\right) \qquad Q_y=-D\left(\frac{\partial^3 w}{\partial x^2 \partial y}+\frac{\partial^3 w}{\partial y^3}\right)$$

where ,

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[10 Marks]

Q3. a) What are the basic assumptions made in the derivation of the equations for the bending of a thin circular plate?

[2 Marks]

b) A circular plate of diameter $\boldsymbol{d_o}$ with a concentric hole of diameter $\boldsymbol{d_i}$ is fixed along the inner boundary and free along the outer boundary. The plate, manufactured with a material having poison ratio of 1/4, is used to be resisted a uniformly distributed vertically downward load of $\boldsymbol{q}$ per unit area. For a value $\frac{d_o}{d_i}=2$,

i) Obtain a quick estimation of the bending moment at the fixed edge, by considering a radial strip as a beam with the loading and end connection as in the plate.

ii) Determine the exact radial moment at the fixed edge.

iii) Compare the results obtained in Part b (i) and Part b (ii) and discuss applicability of the each solution for the design purpose.

Governing equation and the equation for the radial moment of circular plate (with usual notations and sign convention) are given by

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]=\frac{Q}{D} \qquad M_r=-D\left(\frac{d^2 w}{dr^2}+\frac{\nu}{r}\frac{dw}{dr}\right) \qquad M_t=-D\left(\frac{1}{r}\frac{dw}{dr}+\nu\frac{d^2 w}{dr^2}\right)$$

where

$$D=\frac{Et^3}{12(1-\nu^2)}$$

[10 Marks]

Q4. a) Discuss the necessity of ring beams for some of the shell structures.

[2 Marks]

b) A spherical dome, having a radius of $\boldsymbol{R}$, is planned to be used as a roof structure for a sports complex. In order to get the natural lighting, an opening (radius of $\boldsymbol{R_0}$) is planned to be provided along a parallel of the dome at a vertical angle of $\boldsymbol{\beta_0}$. The lightweight glass material, which is supposed to be used to cover the opening, induces a line load of $\boldsymbol{p}$ per unit length at the top edge of the spherical dome. The roof is expected to carry a self-weight of $\boldsymbol{q}$ per unit surface area. The roof structure is simply supported along a parallel at a vertical angle of β $(\beta > \beta_0)$.

i) Identify with justification the locations where ring beams are needed to be provided.
ii) Determine the stress resultants in the spherical roof structure.
Assume that the membrane stresses in a spherical shell (with usual notations and sign convention) are given by

$$\frac{N_\phi}{r_1} + \frac{N_\theta}{r_2} = P_r \qquad P_\phi r r_1 - r_1 N_\theta Cos\phi + \frac{\partial(rN\phi)}{\partial\phi} = 0$$

[10 Marks]

Q5.

a) Discuss the classification of shells with respect to their curvature

[2 Marks]

b) Show that the membrane stresses in a cylindrical shell (*with usual notations and sign convention*) are given by

$$\frac{N_x}{\partial x} + \frac{1}{R}\frac{\partial N_{\phi x}}{\partial \phi} + X = 0 \qquad \frac{1}{R}\frac{\partial N_\phi}{\partial \phi} + \frac{\partial N_{x\phi}}{\partial x} + Y = 0$$

$$\frac{N_\phi}{R} + Z = 0$$

[4 Marks]

c) A semi-circular cylindrical shell made of thin aluminum sheets is proposed to be used as a cantilever roof of a dinning hall as shown in Figure Q5. The length and the radius of the roof are l and a, respectively. The shell is subjected to its self-weight of "w" per unit surface area.
From the membrane theory obtained in Part (b), determine membrane stress resultant in the roof shell structure. *Clearly state any assumption you may made.*

[6 Marks]

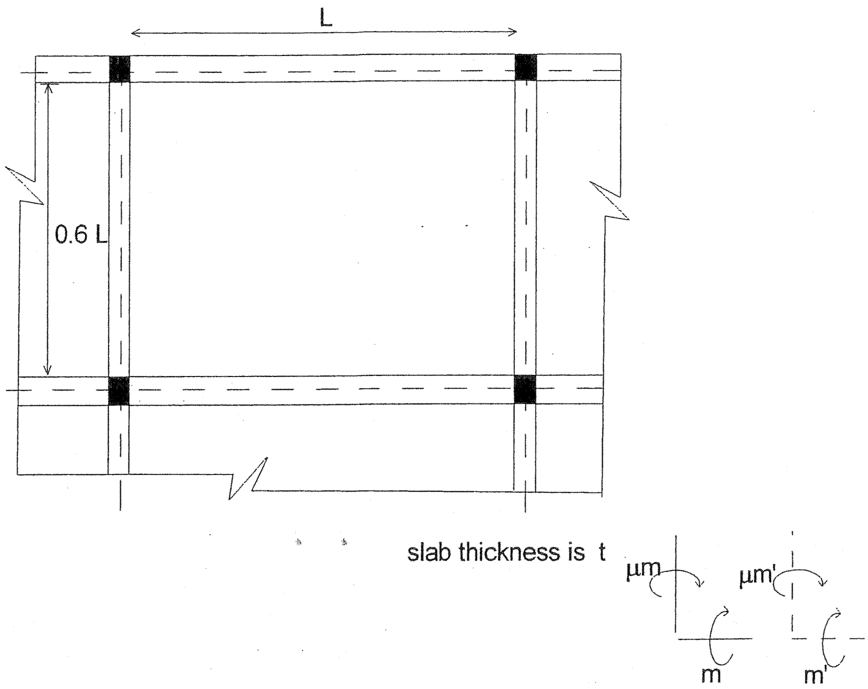

Figure Q1

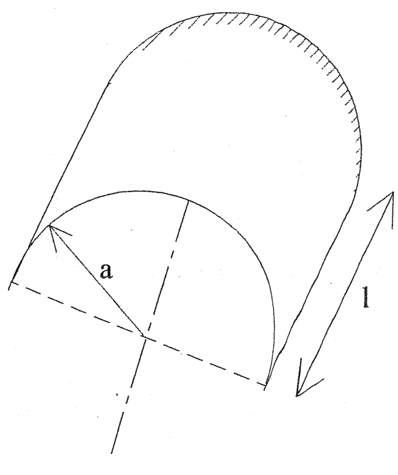

Figure Q5