UNIVERSITY OF RUHUNA

BACHELOR OF SCIENCE GENERAL DEGREE LEVEL II(SEMESTER I) EXAMINATION JUNE/JULY 2013

Subject:PHYSICS
Course Unit: PHY2114

Time: Two hours & 30 minutes

Part II (Essay questions)

Answer at least ONE (01) question from Part B.
Answer FIVE(05) questions only

All symbols have their usual meaning

Part A

1. (a) Inner and outer radii of a cylindrical shell of length l are r_1 and r_2 respectively. The temperatures of the inner and outer surfaces of the shell are maintained at T_1 and $T_2 (T_1 > T_2)$ respectively. The thermal conductivity of the material of the shell is k.

   i. Show that the radial rate of heat transfer through the cylindrical shell is given by

   $$\dot{Q} = \frac{2\pi k l (T_1 - T_2)}{\ln \frac{r_2}{r_1}}$$

   ii. Show that the temperature at a distance $r (r_1 < r < r_2)$ from the axis of the shell is given by

   $$T = T_1 - \frac{(T_1 - T_2)}{\ln \frac{r_2}{r_1}} \ln \frac{r}{r_1}$$

   iii. If $r_1 = 3\,\text{cm}, r_2 = 10\,\text{cm}, T_1 = 150\,^\circ\text{C}, T_2 = 0\,^\circ\text{C}, l = 1\,\text{m}$ and $k = 0.5\,\text{J}\,\text{m}\,\text{s}^{-1}\,\text{K}^{-1}$ calculate $\dot{Q}$.

   iv. Calculate the temperature at $r = 5\,\text{cm}$.

   (b) i. State the Stefan's law of radiation.

   ii. If the solar radiation energy falling on a unit area of the earth per second is $1400\,\text{J}\,\text{m}^{-2}\,\text{s}^{-1}$, calculate the temperature on the surface of the sun.
   Distance from sun to earth $(R) = 1.5 \times 10^{11}\,\text{m}$
   Radius of the sun $(r) = 6.9 \times 10^{8}\,\text{m}$
   The sun's emissivity $(e) = 1$
   $\sigma = 5.67 \times 10^{-8}$

2. (a) Write down the first law of thermodynamics describing each term.

   (b) An ideal gas changes its state

   i. adiabatically from (P_1, V_1, T_1) to (P_2, V_2, T_2).

ii. isothermally from (P_1, V_1, T_1) to (P_3, V_2, T_3).
Indicate the above two processes on a same PV-diagram.
Assume that $V_2 > V_1$)

(c) Show that the slope of an adiabatic curve is γ times slope of an isothermal curve. Here $\gamma = \frac{C_P}{C_V}$.

(d) One mole of an ideal gas ($\gamma = \frac{5}{3}$) at (P_1, V_1, T_1) is capable of executing following processes.
$A \longrightarrow B$: the gas expands isobarically until its volume becomes $2V_1$.
$B \longrightarrow C$: the gas expands isothermally until its volume becomes $3V_1$.
$C \longrightarrow D$: the gas undergoes an adiabatic process.
$D \longrightarrow A$: the gas comes back to the initial state through an isochoric process.

i. Draw a PV diagram for the above cyclic process.
ii. Find pressure and volume after each process in terms of P_1 and V_1.
iii. Obtain an expression for the work done by the gas (W) during this cyclic process.
[When n moles of an ideal gas at temperature T changes its state from (P_1, V_1, T_1) to (P_2, V_2, T_2), the work done by the gas (W) during isothermal and adiabatic processes are given by $W = nRT \ln \frac{r_2}{r_1}$ and $W = \frac{(P_2V_2 - P_1V_1)}{(1-\Gamma)}$ respectively]
iv. If $P_1 = 2 \times 10^5\,\mathrm{N\,m^{-2}}$, $V_1 = 500\,\mathrm{cm^3}$ and $T_1 = 127\,^{\circ}\mathrm{C}$, calculate W.

$R = 8.314$

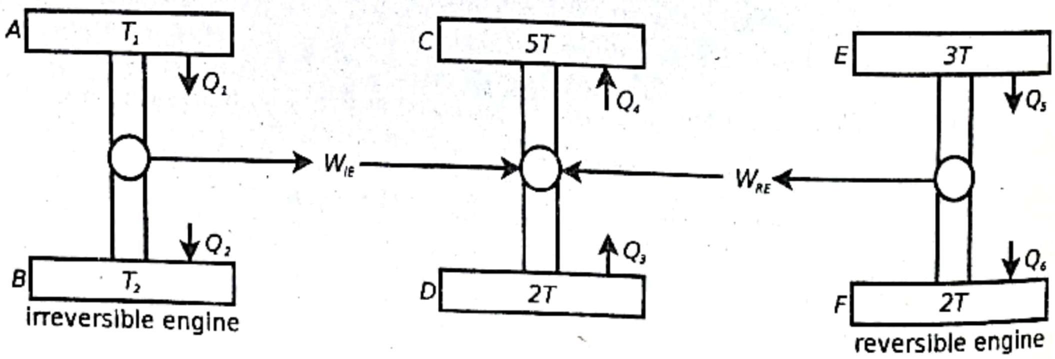
3. (a) Write down main differences between a reversible engine and an irreversible engine.

(b) Define the efficiency, (η) of a heat engine and coefficient of performance, (K) of a refrigerator. Show that the coefficient of performance of a reversible refrigerator is given by

$$K = \frac{T_C}{T_H - T_C}$$

(c) State the Carnot's theorem.

Figure shows an irreversible engine, a reversible refrigerator and a reversible engine operating among six reservoirs $(A, B, C, D, E,$ and $F)$.



i. Calculate the coefficient of performance of the refrigerator.

ii. If $W_{RE} = 200\,\text{J}$, calculate the efficiency of the reversible engine (η_{RE}), Q_5 and Q_6.

iii. If $W_{IE} = 400\,\text{J}$, calculate Q_3 and Q_4.

iv. If the efficiency of the reversible engine (η_{RE}) is equal to the efficiency of the irreversible engine (η_{IE}), does it violate the second law of thermodynamics? Explain your answer.

v. If $\eta_{RE} = \eta_{IE}$ calculate Q_1 and Q_2.

vi. If a heat engine cannot be operated between the reservoirs B and D, and the maximum efficiency of a hypothetical engine operating between the reservoirs A and C is $\frac{2}{3}$, calculate T_1 and T_2 in terms of T.

4. (a) Derive first TdS equation.

(b) Write down second TdS equation.

(c) Using the TdS equations, obtain the heat capacity equation

$$C_P - C_V = -T\left(\frac{\partial V}{\partial T}\right)_P^2\left(\frac{\partial P}{\partial V}\right)_T$$

(d) Define the volume expansion coefficient (β), the isothermal compressibility (K_T) and isothermal bulk modulus (k_T).

(e) Show that

$$C_P - C_V = TV\beta^2 k_T = \frac{TV\beta^2}{K_T}$$

5. (a) Derive the Clausius Clapyron's equation.

(b) Show that the Clausius Clapyron's equation can also be expressed as $\frac{dP}{dT} = \frac{\rho_1\rho_2 L}{T(\rho_1-\rho_2)}$ and $\frac{dP}{dT} = \frac{|S_2-S_1|}{(V_2-V_1)}$.

(c) The atmospheric pressure at the top of mount Hanthana is 0.95 atm. The density of steam and water are $0.595\,\text{kg}\,\text{m}^{-3}$ and $1000\,\text{kg}\,\text{m}^{-3}$ respectively. If the latent heat of vaporization of water is $2.27 \times 10^6\,\text{J}\,\text{kg}^{-1}$, calculate the total energy required to evaporate 10 g of water at 10 °C at the top of mount Hanthana.
$1\,\text{atm} = 1 \times 10^5\,\text{N}\,\text{m}^{-2}$
Specific heat capacity of water (C_W) = $4190\,\text{J}\,\text{kg}^{-1}\,\text{K}^{-1}$

6. (a) Show that the total entropy change of a Carnot cycle is zero. Draw $T-S$ and $P-V$ diagrams for a Carnot cycle. Are the areas bounded by above $T-S$ and $P-V$ curves same? Explain your answer.

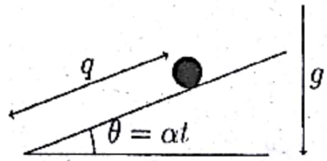
(b) State the Clausius theorem. Hence, show that the entropy is a state function.

(c) A beaker containing 1 kg of ice at −10 °C is placed in thermal contact with a closed container containing 1×10^{-3} kg of steam at 100 °C. Calculate the temperature of the combined system (θ) at the equilibrium state(Hint:$\theta < 0$). Calculate the total entropy change in the above process.

Specific heat capacity of ice $(C_I) = 2095\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Latent heat of fusion of ice $(L_I) = 3.352 \times 10^5\,\mathrm{J\,kg^{-1}}$
Latent heat of vaporization of water $(L_S) = 2.263 \times 10^6\,\mathrm{J\,kg^{-1}}$
Specific heat capacity of water $(C_W) = 4190\,\mathrm{J\,kg^{-1}\,K^{-1}}$

Part B

7. A body with mass m is lying on a smooth horizontal plane. One end of the plane is lifted up at a constant rate as shown in the figure such that the angle θ of the plane with the horizontal increases with time t as $\theta = \alpha t$.



   (a) Show that the Lagrangian of the body expressed in terms of the distance q from the base of the plane is

$$L = \frac{1}{2}m\dot{q}^2 + \frac{1}{2}m\alpha^2 q^2 - mgq\sin\alpha t$$

   (b) Determine the Euler-Lagrangian equations for the system.

8. The potential function of an object is given by $V = k'(x^3 - 2x^2 + x)$. k' is a positive constant and x is the position of the object.

   (a) Find the position of the object when it is at equilibrium.

   (b) Investigate the stability of the object when it is at equilibrium

   (c) Find the period of small oscillations about stable equilibrium positions of the object.

9. A particle of mass m is moving in a central potential $U(r)$.

   (a) Show that the radial equation of the mass is given by

$$m\ddot{r} - \frac{l^2}{mr^3} = -\frac{dU(r)}{dr}$$

   where r is the radial distance and l is the angular momentum.

   (b) Show that the radial equation in part(a) can be written as

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{l^2}\frac{dU(r)}{du}$$

   where $u = \frac{1}{r}$