PART B **Time: 01 hour and 30 minutes**

Answer only four(04) questions

25 marks for each questions

1. Use the data in the following table to answer the questions below.

number	Luminosity / $L_\odot$	Spectral Type	Type of star
1	10^4	B	main sequence
2	0.01	B	white dwarf
3	0.01	M	main sequence
4	10^4	M	Giant
5	1	G	main Sequence

(a) Sketch the locations of stars given in the above table in a HR diagram. (Note: You should label the axis and stars with their relevant numbers)

(05 Marks)

(b) Estimate the mass of main sequence stars using the luminosity values given in the table.

(05 Marks)

(c) Relate your results in part (b) to the positions of those stars in the HR diagram.

(03 Marks)

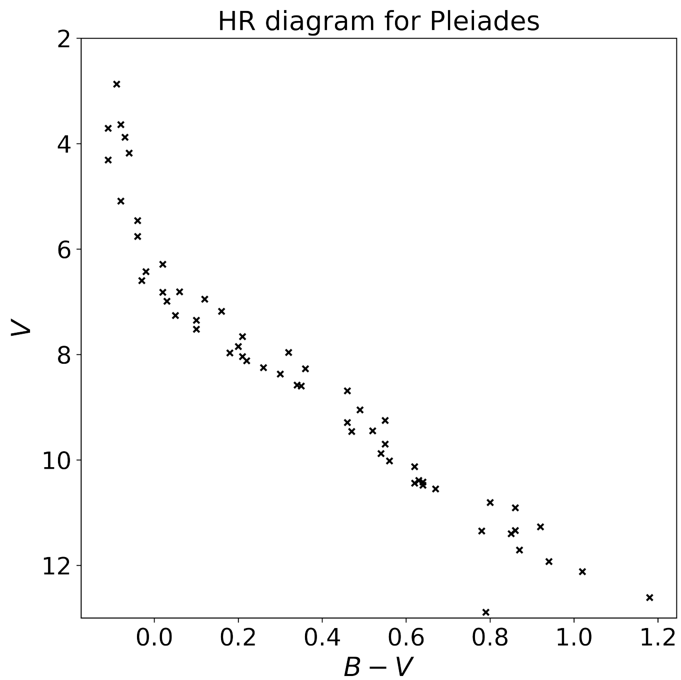
(d) Even though stars numbered 3 and 4 belong to the same spectral class M, star 4 is 10^4 times luminous compared to star 3. Justify the statement.

(05 Marks)

(e) Predict the fate of star 1 and 3 and explain their evolutionary process, briefly.

(07 Marks)

2. Following figure is a HR diagram made using data from 57 stars that belongs to the Pleiades (M45) star cluster. V (y axis) is the apparent magnitude measured using the V (yellow) filter and B is the apparent magnitude measured using the B(blue) filter.



(a) Discuss the advantage of using star clusters to plot HR diagrams.

(05 Marks)

(b) Assume that one measures temperatures and calculate the absolute magnitudes of stars and decides to change the x and y axes of the above HR diagram into temperature and absolute magnitudes, respectively. Will this change affect the shape of HR diagram significantly? Justify your answer.

(05 Marks)

(c) Is the Pleiades star cluster open or globular? Discuss the reason for your choice.

(03 Marks)

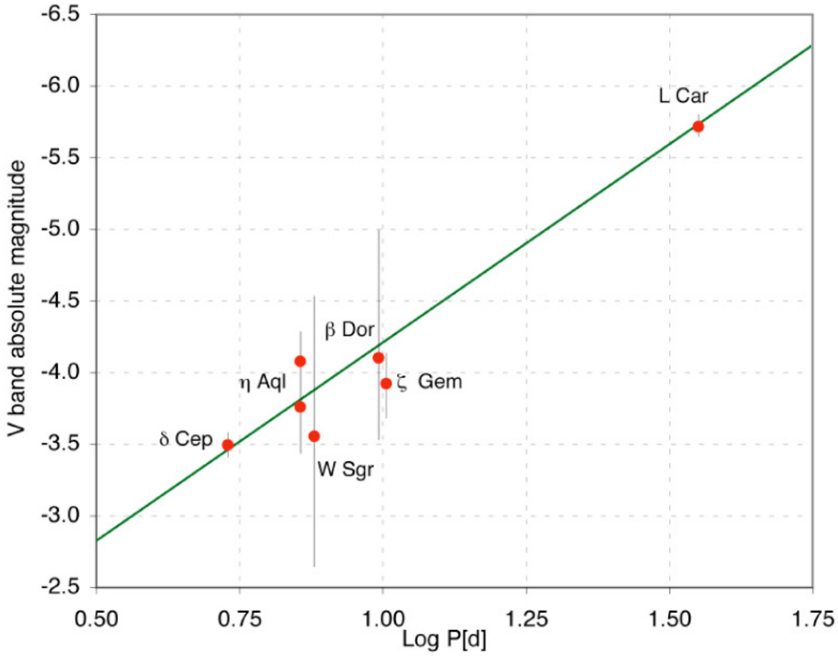
(d) Results from the European astrometric satellite, HIPPARCOS, gave a distance of 116 parsecs to the Pleiades. Calculate the absolute magnitude (M_v) of the brightest star ($m_v = 2.87$) in the cluster.

(05 Marks)

(e) Our galaxy contains about 200 globular clusters. Assuming that you have the data to plot HR diagrams for all the clusters in Milky Way, suggest a method to filter those 200 globular clusters.

(07 Marks)

3. Following plot is the Period-Luminosity (P-L) relation in the V band, as deduced from the interferometric observations of Cepheids and the HST parallax measurement. The straight line is the fitted P-L relation.



(a) Discuss the importance of Cepheid variable stars in discovery of nearby galaxies.

(07 Marks)

(b) Identify the difference between RR-Lyrae stars and Cepheid variable stars and decide which type is more suitable for measuring distances to nearby galaxies.

(05 Marks)

(c) An Astronomer observed a Cepheid variable star and recorded that the period of the star as 5.6 days (d). If the apparent magnitude of the star in V band (m_v) is 15.5, Estimate the distance to the star and decide whether the star belongs to the Milky Way.

(07 Marks)

(d) One could also use Type Ia supernovae to measure the distances to galaxies. What are the advantages of using Type Ia supernovae compared to Cepheid variables?

(06 Marks)

4. Consider a surface of a sphere defined by
$ds^2 = R^2\, d\theta^2 + R^2 Sin^2(\theta)\, d\Phi^2$, where R is a constant.

   (a) Write down $[g_{\mu\nu}]$ and $[g^{\mu\nu}]$.

   (02 Marks)

   (b) Connection coefficients (Christoffel symbols) are defined by $\Gamma^{\sigma}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho}(\frac{\partial g_{\rho\nu}}{\partial x^{\mu}} + \frac{\partial g_{\mu\rho}}{\partial x^{\nu}} - \frac{\partial g_{\mu\nu}}{\partial x^{\rho}})$.
   Calculate $\Gamma^{\theta}_{\phi\phi}$ and $\Gamma^{\phi}_{\phi\theta} = \Gamma^{\phi}_{\theta\phi}$, which are the only nonzero components.

   (09 Marks)

   (c) Geodesic equation is defined as, $\frac{DU^{\alpha}}{d\lambda} = \frac{d^2x^{\alpha}}{d\lambda^2} + \Gamma^{\alpha}_{\gamma\beta}\frac{dx^{\beta}}{d\lambda}\frac{dx^{\gamma}}{d\lambda} = 0$.
   Starting from the geodesic equation verify that the curve on the surface of a sphere that is defined by $\theta = \frac{\pi}{2}$ and $0 \leq \phi \leq 2\pi$ is a geodesic.

   (07 Marks)

   (d) Calculate the Riemann curvature tensor $R^{\theta}{}_{\phi\theta\phi}$
   Riemann curvature tensor is defined as, $R^{l}{}_{ijk} = \frac{\partial \Gamma^{l}_{ik}}{\partial x^{j}} - \frac{\partial \Gamma^{l}_{ij}}{\partial x^{k}} + \Gamma^{m}_{ik}\Gamma^{l}_{mj} - \Gamma^{m}_{ij}\Gamma^{l}_{mk}$

   (07 Marks)

5. Schwarzschild black holes can be described using Schwarzschild metric given by,
$ds^2 = (1 - \frac{2GM}{c^2 r})c^2 dt^2 - \frac{dr^2}{1-\frac{2GM}{c^2 r}} - r^2 d\theta^2 - r^2 sin^2\theta\, d\Phi^2$
and orbital motion equation of the Schwarzschild space time is given as,
$\left(\frac{dr}{d\tau}\right)^2 + \frac{J^2}{m^2 r^2}\left(1 - \frac{2GM}{c^2 r}\right) - \frac{2GM}{r} = c^2\left[\left(\frac{E}{mc^2}\right)^2 - 1\right]$
where,
$\frac{J}{m} = r^2\frac{d\phi}{d\tau}$ and $R_s = \frac{2GM}{c^2}$
$J =$ angular momentum

   (a) i. Consider the motion of an observer falling freely radially down towards the center of a black hole. If he had started at rest from infinity, show that the proper time between two arbitrary points r_1 and r_2 ($r_2 < r_1$) is,
   $\tau = \frac{2}{3c\sqrt{R_s}}\left(r_1^{\frac{3}{2}} - r_2^{\frac{3}{2}}\right)$

   ii. An observer starts to fall freely from infinity towards a three solar mass black hole of Schwarzschild radius $R_s = 9$ km. How long does it take the observer to fall from $r_1 = 25$ km to the event horizon?

   (10 Marks)

   (b) Assume that a distant observer A at rest looks at an object falling radially into a black hole.

   i. As measured by A, show that the time it takes for a light signal emitted by the object to reach for this observer is,
   $t_2 - t_1 = \frac{r_2 - r_1}{c} + \frac{R_s}{c}\, ln\left(\frac{r_2 - R_s}{r_1 - R_s}\right)$

$\left(Hint : \int \frac{dr}{1-\frac{R_s}{r}} = R_s\, ln(r - R_s) + r\right)$

ii. What would be the journey time under the absence of a central mass.

iii. Show that A will never see the object reaching to the event horizon.

(15 Marks)

6. Friedmann equation is given as,

$$\left[\frac{1}{R}\frac{dR}{dt}\right]^2 = \frac{8\pi G}{3}\left[\rho_{m,0}\left(\frac{R_0}{R(t)}\right)^3 + \rho_{r,0}\left(\frac{R_0}{R(t)}\right)^4 + \rho_\Lambda\right] - \frac{kc^2}{R^2}$$

Where R is the scaling factor, k is the curvature parameter and ρ_m, ρ_r, ρ_Λ are matter, radiation and dark energy densities, respectively.

Assume that the time started from the Big Bang where the scale parameter(R) is zero and scale factor $R = R_0$ at present time $t = t_0$.

(a) Summarize evidences that you have to claim for the validity of the cosmological principle.

(08 Marks)

(b) Discuss how the matter, radiation and dark energy densities have changed with the expansion of the universe since the Big Bang.

(03 Marks)

(c) Use the Friedmann equation to derive a relationship between scaling parameter (R) and the time (t) for a flat, matter dominated universe.

(08 Marks)

(d) Relate the Hubble parameter ($H(t) = \frac{1}{R}\frac{dR}{dt}$) to the critical density of the universe and discuss the role of critical density in deciding the geometry of the universe.

(06 Marks)