UNIVERSITY OF RUHUNA
BACHELOR OF SCIENCE (SPECIAL) DEGREE LEVEL II (SEMESTER II)
EXAMINATION – AUGUST 2021

SUBJECT: PHYSICS
COURSE UNIT: PHY4144 TIME: 2 hours

Answer FIVE (04) Questions only.
(All symbols have their usual meaning)

1. Let $f(x) = x^5 + x^4 - 3$
   a. Prove that f has one root r in the interval $[1,2]$
   b. Compute two steps of the bisection method on $[1,2]$. That is, with $x_1 = 1.5$, find x_2 and x_3.
   c. How many steps of the bisection method are required to approximate the root to within 10^{-100}?
   d. Compute two steps of Newton's method with $x_0 = 1$.
   e. Apply two steps of the second method with initial guesses $x_0 = 1, x_1 = 2$.

2. Consider the quadrature rule

$$\int_0^1 f(x)dx = w_1 f(0) + w_2 f'(x_2)$$

   a. Show that this rule gives the highest possible degree of accuracy when

$$w_1 = 1, w_2 = \frac{1}{2}, x_2 = \frac{1}{3}$$

   b. Use the quadrature rule in (a.) to approximate

$$\int_0^1 \frac{1}{x^2 + e^x} dx$$

3. a. Consider the following initial value problem

$$y' = y^2, \; y(0) = 1$$

   i.) Use one iteration of fourth order Runge-Kutta method to approximate $y(0.2)$
   ii.) Use two iterations of Heun's method to approximate y(0.2)

b. One way to calculate π is to use the identity $\tan^{-1}(1) = \frac{\pi}{4}$ together with numerical quadrature to evaluate

$$\tan^{-1}(1) = \int_0^1 \frac{1}{1+x^2}\,dx$$

i.) Use the composite Simpson's rule (with $n = 4$) to approximate π

4. a. The following is a table of values for a

x	0	1	3	4
$f(x)$	1.5	0.0	1.0	2.0

i. Write a polynomial that interpolates these data.
ii. Approximate the value of $f(2)$
iii. If 20 tabulated values of $f(x)$ were given instead, what would be the degree of the polynomial interpolating all 20 points?

b. Find an interpolating polynomial to the four points using Newton's divided difference method.

5.

a. Consider the following linear system

$$\begin{aligned} x_1 + x_2 + x_3 &= 5 \\ x_1 + 3x_2 + x_3 &= 2 \\ 3x_1 + x_2 + x_3 &= 4 \end{aligned}$$

i. Reorder the equations so that Jacobi iteration will converge to the exact solution
ii. Carry out two iterations with starting vector $x = (0,0,0)$

b. Derive the first-order derivative formulas of a function $f(x)$ and list the order of their error term used in

i. forward difference method

ii. central difference method