



University of Ruhuna
Faculty of Fisheries and Marine Sciences & Technology

B.Sc. Hons in Fisheries and Marine Sciences Degree
Level I (Semester I) – September 2018

Module Name: Mathematics
Exam date: 2018-09-12

Module Number: FDN1142
Time: Two (02) hours

Answer ALL Questions. Calculators will be provided.

1. a) Find the following limits:

(i) $\lim_{x \rightarrow 3} \frac{x^2 + x - 12}{2x^2 - 5x - 3}$,

(ii) $\lim_{x \rightarrow 25} \frac{x - 25}{\sqrt{x} - 5}$.

b) A colony of viruses can be modelled by the rule

$$N(t) = \frac{2t}{(t + 0.5)^2} + 0.5,$$

where $N(t) \times 10^5$ is the number of viruses on a nutrient plate t hours after they started multiplying.

(i) How many viruses are present initially?

(ii) Find $\frac{dN(t)}{dt}$.

(iii) At what **rate** would the virus numbers be changing after 10 hours?

c) Find the first derivative $f'(x)$ of $f(x) = \frac{\sin(x)}{e^{2x}}$ and hence find the gradient at the point where $x = 0$.

2. a) Determine the stationary points of the function

$$f(x) = \frac{2x^3}{3} + \frac{3x^2}{2} - 2x + 4$$

and classify them as maxima or minima using the second order derivative $f''(x)$.

b) The curve $y = ax^2 + bx + c$ passes through the point $(0, 8)$ and has a stationary point at $(1, 5)$. Find the values of a, b and c .

Contd...

3. a) The van der Waals equation for n moles (n is a constant) is given by

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT.$$

- (i) Find P in terms of V and T .
(ii) Find the partial derivatives $\frac{\partial P}{\partial V}$ and $\frac{\partial P}{\partial T}$.
(iii) Write down the expression for total differential dP in terms of V, T, dV and dT .
(iv) Show also that

$$\frac{\partial}{\partial V} \left(\frac{\partial P}{\partial T} \right) = \frac{\partial}{\partial T} \left(\frac{\partial P}{\partial V} \right).$$

- b) Evaluate $\int \frac{1}{\sqrt{x+1}} dx$ using the substitution $t = \sqrt{x}$.
c) Find the constants A and B such that

$$\frac{3x+4}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}.$$

Hence, evaluate

$$\int \frac{3x+4}{(x-2)(x+3)} dx.$$

4. a) Use **integration by parts** to evaluate the integral

$$\int_1^2 te^{2t} dt.$$

- b) Show that the differential equation

$$(2xy - 3x^2) + (x^2 - 2y) \frac{dy}{dx} = 0$$

is exact and find its solution.

- c) Solve the differential equation

$$\tan(\theta) \frac{dx}{d\theta} = x; \text{ where } x = 2 \text{ when } \theta = \pi/6,$$

using the **method of separation of variables**.

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