UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: August 2015

Module Number: IS1401 **Module Name: Mathematical Fundamentals for Engineers**

[Three hours]

[Answer **all questions,** each question carries fourteen marks]

Write your answers for **PART-A** and **PART-B** in separate booklets

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PART-A

Q1. a) i.) If $z = x + iy$ is a complex number, where x and y are real numbers, briefly explain what is meant by the modulus and the argument of z.

ii.) Given that

$$\frac{1}{z} = (1-i)^2 + \frac{3\sqrt{3}-i}{2-\sqrt{3}i}.$$

Express the complex number z in the form $x + iy$.

Find the modulus and the argument of z.

[3 Marks]

b) i.) Find all the roots of $x^6 - 64 = 0$, in the form $x + iy$, where x and y are real numbers.

ii.) Show all the six roots of the above equation on an Argand diagram.

[6 Marks]

c) Express $\cos 6\theta$ in terms of $\cos\theta$ and $\sin\theta$.

Hence, solve the equation

$$16x^6 - 24x^4 + 9x^2 - 1 = 0$$

[5 Marks]

Q2. a) i.) Explain what is meant by the statement, function '$f(x)$ is continuous at c'.

ii.) Prove that if $f(x)$ is finitely differentiable at c then $f(x)$ is continuous at c.

Is converse of the above true? Justify your answer.

iii.) Show that the following function is not continuous at $x = 0$.

$$f(x) = \frac{2\,e^{1/x} - 1}{e^{1/x} + 2}$$

[7 Marks]

b) By using the rules of limits evaluate,

i.) $\lim_{x\to 1}\left(\frac{x^n + x^{n-1} + \cdots + x - n}{x-1}\right)$ ii.) $\lim_{x\to 0}\left(\frac{1-\cos^3 x}{\sin^2 x}\right)$

[4 Marks]

c) i.) Briefly explain what is meant by a 'one to one function'.

ii.) Show that if $f(x)$ and $g(x)$ are one to one on $\mathbb{R}$ then $(f \circ g)(x)$ is also one to one.

[3 Marks]

Q3. a) i.) State the Mean Value Theorem.

ii.) If f and g are continous in $[a, b]$ and differentiable in (a, b) with $g'(x) \neq 0$ for all $x \in (a, b)$, prove that there exist $c \in (a, b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

iii.) Use Mean Value Theorem to show that

$$\sin x < x < e^x - 1$$

Hence, deduce that

$$\frac{\sin x}{x} < \frac{e^x}{1 + x}$$

iv.) If $f(x)$ is a twice differentiable function and satisfies $f(2b) = 2f(b)$ and $f''(x) < 0$ for all $x \geq 0$. If $b > a > 0$, by considering two applications of mean value theorem on the interval $[a, b]$ and $[a + b, 2b]$, show that

$$f(a + b) \geq f(a) + f(b)$$

[9 Marks]

b) i.) State and prove the Euler's theorem on homogeneous function of two variables.

ii.) If

$$u = \cos^{-1}\left(\frac{x^4 + y^4}{x - y}\right)$$

Show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + 3\cot u = 0$$

[5 Marks]

PART B

Q4. a) Briefly explain the following topics by giving an example for each item.

i.) Equal matrices
ii.) Transpose of a matrix
iii.) Skew symmetric matrix
iv.) Hermitian matrix

[2 Marks]

b) If A and B are two matrices conformal for product AB, then show that $(AB)^T = B^T A^T$, where A^T and B^T are the transpose matrices.

[3 Marks]

c) i.) If A and B are non-singular matrices of the same order and A^{-1} and B^{-1} are the inverse matrices of A and B respectively, show that

$$(AB)^{-1} = B^{-1}A^{-1}.$$

ii.) By the method of matrix inversion, solve the system.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & 7 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x & u \\ y & v \\ z & w \end{bmatrix} = \begin{bmatrix} 9 & 2 \\ 52 & 15 \\ 0 & -1 \end{bmatrix}$$

[5 Marks]

d) Determine the values of a and b for which the system

$$\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$$

i.) has a unique solution,
ii.) has no solution and,
iii.) has infinitely many solutions.

[4 Marks]

Q5. a) i.) Briefly explain what is meant by 'Unit vector' and 'Position vector'.

ii.) Show that the three points, whose position vectors are $\left(2\underline{i} + 3\underline{j} - 4\underline{k}\right)$, $\left(\underline{i} - 2\underline{j} + 3\underline{k}\right)$, and $\left(-7\underline{j} + 10\underline{k}\right)$ are collinear.

iii.) A particle is displaced from the point whose position vector is $\left(5\underline{i} - 5\underline{j} - 7\underline{k}\right)$ to the point whose position vector is $\left(6\underline{i} + 2\underline{j} - 2\underline{k}\right)$ under the action of the constant forces $\left(10\underline{i} - \underline{j} + 11\underline{k}\right)$, $\left(4\underline{i} + 5\underline{j} + 6\underline{k}\right)$ and $\left(-2\underline{i} + \underline{j} - 8\underline{k}\right)$. Find the work done by the forces.

[5 Marks]

b) A rigid body is spinning with angular velocity 6 radians/sec about an axis OR where R is $\left(2\underline{i} + 2\underline{j} + \underline{k}\right)$ and O is the origin. Find the velocity of the point $\left(-3\underline{i} + -2\underline{j} + \underline{k}\right)$ on the body.

[4 Marks]

c) i.) Evaluate $grad\ \phi$ if $\phi = \log(x^2 + y^2 + z^2)$

ii.) Given the vector field,

$$\underline{V} = (x^2 - y^2 + 2xz)\underline{i} + (xz - xy + yz)\underline{j} + (x^2 + y^2)\underline{k}.$$

Find $curl\ \underline{V}$.

Show that the vectors given by $curl\ \underline{V}$ at $P_0(1,2,3)$ and $P_1(2,3,12)$ are orthogonal.

[5 Marks]