UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 2 Examination in Engineering: December 2015

Module Number: ME 2201 **Module Name: Engineering Mechanics**

[Three Hours]

[Answer **all questions**, each question carries **ten** marks]

Q1. a) Define the terms; Configuration Space, Co-ordinate Transformation, and Degrees of Freedom (DOF).

[3 Marks]

b) Explain mathematically the concept of "holonomic constraints"

[2 Marks]

c) Two point masses P_1 and P_2 are moving in three-dimensional Euclidean space such that P_1 moves on a sphere and the distance between P_1 and P_2 remains fixed. Specify the configuration space and a suitable set of coordinates for the system. What is the DOF of the system?

[5 Marks]

Q2. A particle P of mass m is constrained to move on the surface of a sphere of radius r and origin O as shown in Figure Q2. Except for the constraints, no other external forces act on the particle. Let e be a frame with origin coinciding with O and fixed with respect to the sphere. The representation of the point P with respect to frame e is x.

a) Show that $x^T x = r^2$.

[2 Marks]

b) Show that the velocity of the particle is always tangential to sphere (orthogonal to the position x, i.e. $x^T \dot{x} = 0$).

[3 Marks]

c) Differentiating the constraint $x^T \dot{x} = 0$, show that the motion of the particle in the frame e is described by

$\ddot{x} = \frac{1}{m} f_c(t) = -\frac{\|\dot{x}\|^2}{r^2} x$, where $f_c(t)$ is the representation of the constraint force in the e frame. (Hint: The constraint that keeps a particle on the surface of a sphere should act in the radial direction).

[5 Marks]

Q3. a) A particular moving frame is said to be *inertial* with respect to a fixed frame. Explain the aforementioned statement mathematically.

[2 Marks]

b) Explain mathematically that an observer moving with a translating frame experiences an additional force (*apparent force*); in addition to the *force felt in the Earth coordinate system*.

[3 Marks]

c) Briefly explain why a person standing on a scale inside an elevator sees his or her weight doubled as the elevator accelerates up at a rate g and sees the weight reduced to zero if the elevator decelerates at a rate of g, where g is acceleration due to gravity.

[5 Marks]

Q4. For a particle moving in 2-dimension space, write down explicitly the motion of the particle as observed in the moving frame $b(t) = [e_r(t) \quad e_\theta(t)]$ (refer to Figure Q4). Also write down the apparent forces acting on the particle as observed by an observer in the moving frame. If the particle is constrained to move in a circle, write down the equations of motion and the constraint forces.

[10 Marks]

Q5. The three frames $a(t)$, $b(t)$, $c(t)$ are related to a fixed frame e as shown in Figure Q5.

Let $a(t) = eR_1(\psi)$, $b(t) = eR_2(\phi)$, and $c(t) = eR_3(\theta)$.

Show the followings.

$$R_1(\psi)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\psi & -\sin\psi\\ 0 & \sin\psi & \cos\psi\end{bmatrix},\ R_2(\varphi)=\begin{bmatrix}\cos\phi & 0 & \sin\phi\\ 0 & 1 & 0\\ -\sin\phi & 0 & \cos\phi\end{bmatrix},\ R_3(\theta)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$$

and

$$R_1^T\dot{R}_1=\hat{\Omega}_1=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & -\dot{\psi}\\ 0 & \dot{\psi} & 0\end{bmatrix},\ R_2^T\dot{R}_2=\hat{\Omega}_2=\begin{bmatrix}0 & 0 & \dot{\varphi}\\ 0 & 0 & 0\\ -\dot{\varphi} & 0 & 0\end{bmatrix},\ R_3^T\dot{R}_3=\hat{\Omega}_3=\begin{bmatrix}0 & -\dot{\theta} & 0\\ \dot{\theta} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

[10 Marks]

Q6. a) Draw the free body diagram for the quarter car model shown in Figure Q6 with usual notations.

[4 Marks]

b) Derive equations of motion for the system.

[6 Marks]

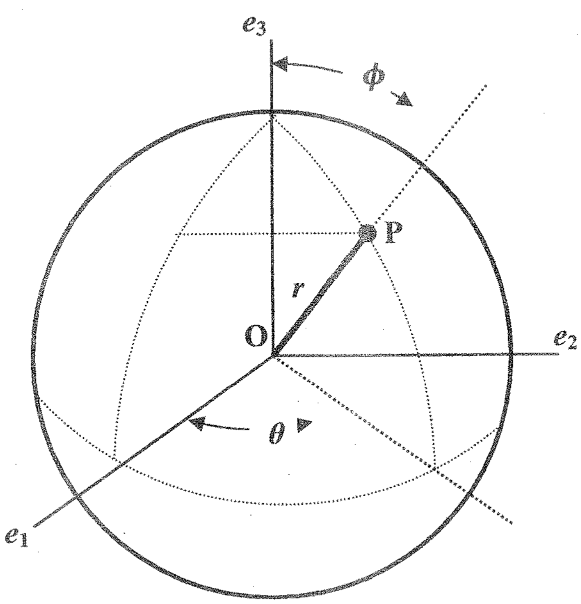

Figure Q2

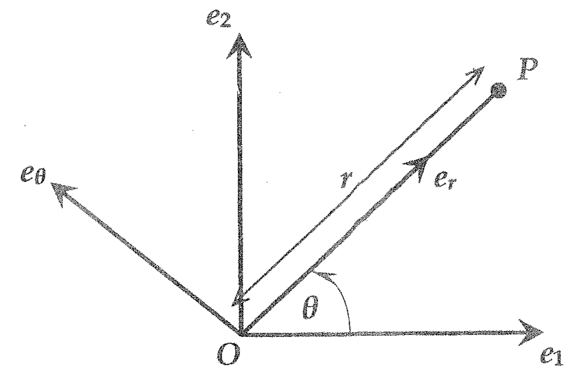

Figure Q4

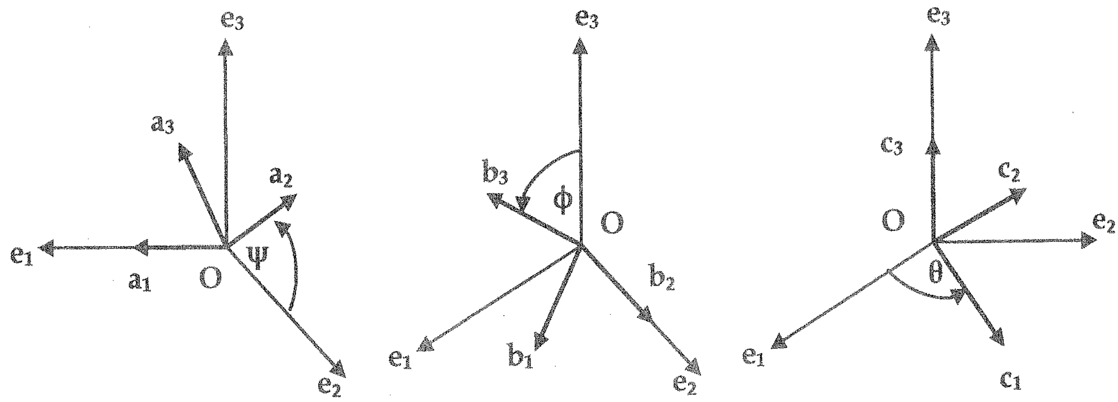

Figure Q5

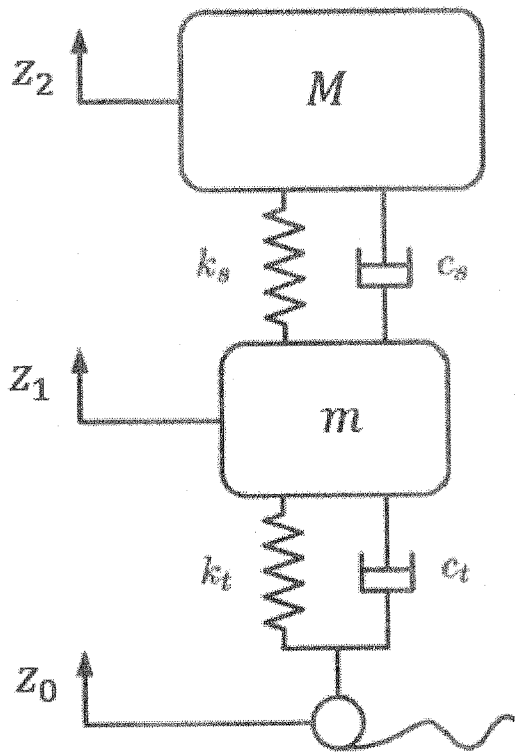

Figure Q6