UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: August 2015

Module Number: EE3205 **Module Name: Signals and Systems**

[Three Hours]

[Answer **all questions**, each question carries 10 marks]

All the symbols have their usual meanings.

Q1 a) i) Explain how a continuous-time signal is classified as either a power signal or an energy signal.

ii) Sketch the following signal and determine whether it is a power signal or an energy signal.

$$x(t) = A\left[u(t+a) - u(t-a)\right]$$

Assume that $0 < A,\ a < \infty$ and $u(t)$ denotes the unit-step function.

[4.0 Marks]

b) Consider the discrete-time system with input $x[n]$ and output $y[n]$ related by

$$y[n] = x[n]\, x[n-2].$$

i) Is this system linear?

ii) Is this system memoryless?

Note : Justify your answers clearly.

[4.0 Marks]

c) Explain how the cross-correlation between two signals is used to identify the received signal at the receiver in a digital communication system.

[2.0 Marks]

Q2 a) i) $F(x)$ is a function with a period 2π. Express $F(x)$ in terms of an infinite sum of sine terms and cosine terms using coefficients a_0, a_m and b_m.

ii) Using the expression obtained in part i), show that

$$b_m = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\, \sin(mx)\, dx.$$

[5.0 Marks]

b) Consider the following periodic signal.

$$f(x) = \begin{cases} -1 & -\pi \le x \le \frac{-\pi}{2} \\ \frac{1}{2} & \frac{-\pi}{2} \le x \le \frac{\pi}{2} \\ 1 & \frac{\pi}{2} \le x \le \pi \end{cases}$$

i) Sketch the signal $f(x)$ for $-2\pi \le x \le 2\pi$.

ii) Determine the Trigonometric Fourier series of the periodic signal $f(x)$ using at least five terms.

[5.0 Marks]

Q3 a) Consider the following expressions.

$$\operatorname{sgn}(t) = \begin{cases} -1 & t > 0 \\ 0 & t = 0 \\ 1 & t < 0 \end{cases}$$

$$f_a(t) = e^{-at}u(t) - e^{at}u(-t)$$

$$y(t) = \frac{1}{2}\delta(t) + \frac{1}{2}\operatorname{sgn}(t)$$

i) Write the expressions for the Fourier transform and Inverse Fourier Transform using usual symbols.

ii) Show that $\operatorname{sgn}(t)$ is approximated using the function $f_a(t)$ when $a \to 0$.

iii) Determine the Fourier transform of the signal $f_a(t)$.

iv) Hence or otherwise determine the Fourier transform of the signal $\operatorname{sgn}(t)$ for $\omega \neq 0$.

v) Using the results obtained in part iii) and iv), determine the Fourier transform of the function $y(t)$.

[6.0 Marks]

b) Consider the following aperiodic signal.

$$f(t) = \begin{cases} 1 & |t| \leq a \\ 0 & |t| > a \end{cases}$$

i) Determine the Fourier transform of $f(t)$.

ii) Sketch the time domain and frequency domain waveforms of the signal $f(t)$.

[4.0 Marks]

Q4 a) Comment on the response of a Linear Time-Invariant (LTI) system for a general complex exponential input.

[1.0 Mark]

b) Consider the LTI system with an impulse response of h(t) shown in Figure Q4.

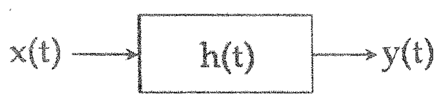

Figure Q4: LTI system

Show that a complex exponential input with any complex number s gives an ideal eigen function for an LTI system.

Hint : Convolution Integral

[1.5 Marks]

c) i) Determine the Laplace transform $X(s)$ of the signal

$$x(t) = 4e^{-3t}\,u(t) - 3e^{-2t}\,u(t).$$

ii) Sketch the pole-zero plot clearly showing the region of convergence in the complex plain.

[4.5 Marks]

d) i) Determine the Laplace transform and the region of convergence of the signal

$$x(t) = -Ae^{-Bt}\,u(-t)$$

where $A,\ B > 0$.

ii) Hence, determine the continuous-time signal of

$$X(s) = \frac{3}{2s+5} \quad \text{for} \quad \mathrm{Re}\{s\} < -\frac{5}{2}.$$

[3.0 Marks]

Q5 a) i) State the Nyquist sampling theorem for continuous-time signals.

ii) Explain why it is necessary to make a band-limited signal before sampling a continuous-time signal.

[2.0 Marks]

b) A lowpass signal $x(t)$ has a spectrum

$$X(f) = \begin{cases} 1 - \left| \dfrac{f}{200} \right| & |f| \le 200 \\ 0 & \text{otherwise} \end{cases}.$$

i) Assume that $x(t)$ is ideally sampled at a sampling frequency $f_s = 300$ Hz. Sketch the spectrum of the sampled signal of $x(t)$ for $|f| \le 600$.

ii) Repeat part i), if $f_s = 400$ Hz.

iii) How do you relate the results obtained in part b i) and b) ii) with the Nyquist sampling theorem stated in part a) i) ?

[5.0 Marks]

c) "The natural sampling is rarely employed in practice. Instead the other practical sampling technique i.e. flat-top sampling is employed in practice."
Explain the difference between natural sampling and flat-top sampling.

[3.0 Marks]