UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: August 2015

Module Number: ME3303 **Module Name: Modelling of Dynamic Systems**

[Three Hours]

[Answer **all questions**, each question carries ten marks]

A partial table of Laplace transformation pairs is given in page number 5. You may make additional assumptions where necessary, but clearly state them in your answers. Symbols stated herein denote standard parameters.

Q1 a) Use Laplace transformation to find the solutions of following ordinary differential equations considering relevant initial conditions.

i) $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+5y=0;\ y(0)=1,\ \dot{y}(0)=0$

ii) $\dfrac{d^2y}{dt^2}-2\dfrac{dy}{dt}+y=e^t;\ y(0)=-2,\ \dot{y}(0)=-3$

[5.0 Marks]

b) The current $i(t)$ in an electrical circuit is given by,

$$\frac{d^2i}{dt^2}+2\frac{di}{dt}=0;\ \text{if } 0<t<10$$

$$=1\ ,\ \text{if } 10<t<20$$
$$=0\ ,\ \text{if } t>20$$

Determine the current as a function of 't' considering zero initial conditions.

[5.0 Marks]

Q2 a) The mathematical model of a linear system is given by,

$$\ddot{x}(t)+2\zeta\dot{x}(t)+\omega^2x(t)=\alpha y(t)$$
$$\dot{y}(t)+\beta y(t)+\alpha\dot{x}(t)=v(t)$$

The input is $v(t)$ and output is $x(t)$. Construct a block diagram of the system and find the system transfer function considering zero initial conditions.

[5.0 Marks]

b) The mechanical system shown in Figure Q2(b) is at rest for t = 0. The input force $u(t)$ is given at $t=0$. The displacement $x(t)$ which is the output of the system is measured from the equilibrium position. Obtain the transfer function $X(s)/U(s)$ of the system.

[5.0 Marks]

Q3 a) The characteristic differential equation for second order system is given by,
$\ddot{y}(t) + a\dot{y}(t) + by(t) = ku(t)$.

i) Obtain the transfer function of the system considering zero initial conditions and find the poles of the system in terms of system parameters.

ii) Find the damping ratio (ζ) and natural frequency (ω_n) in terms of system parameters.

iii) If a unit step input with magnitude 'A' is given to the system find the response y(t) of the second order system.

iv) Comment on the stability and response y(t) if

α) $\zeta = 0$

β) $0 < \zeta < 1$

γ) $\zeta = 1$

Hint: *Use Laplace table to obtain solutions.*

[8.0 Marks]

b) A second order control system is given by the transfer function,

$$G(s) = \frac{24}{s^2 + 3s + 43}$$

Determine the natural undammed frequency ω_n and damping ratio ζ.

[2.0 Marks]

Q4 a) In the electrical circuit shown in Figure Q4(a), the strength of current source $i(t)$ and the voltage $v_2(t)$ across the inductor L_1 are the input and the output of the system respectively. Assuming all components are linear, determine the state-space model of the system considering inductors current (i_{L_1}, i_{L_2}) and capacitor voltage as state variables.

[5.0 Marks]

b) In the M-S-D system shown in Figure Q4(b), the external force f(t) and the displacement 'y' are the input and the output of the system respectively. Assuming all components are linear, determine the state-space model of the system considering state variables as x (Displacement of mass 'm'), $\dot{x}$ (Velocity of mass 'm') and y.

[5.0 Marks]

Q5 A system is defined by,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 2 \\ -2 & 2 & 2 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

a) Obtained an expression for the characteristic equation of the system.

[2.0 Marks]

b) Determine the system eigenvalues and comment on the stability of the system.

[2.0 Marks]

Q5 is continued to next page....

c) Derive the transfer function matrix of the system.

[6.0 Marks]

Q6 The governing equation of a simple pendulum of length L and total mass M with an oscillated angle of θ is given by,

$$\ddot{\theta} + \frac{3C}{ML^2}\dot{\theta} + \frac{3g}{2L}\sin\theta = 0.$$

Where C is the viscous frictional torque due to air resistance and bearing resistance acting on the pendulum and, g is the acceleration due to gravity.

a) By using state space representation, obtain the dynamic system model of the system.

[2.0 Marks]

b) Rearrange the system to find the equilibrium solutions, analytically.

[2.0 Marks]

c) Find all the equilibrium solutions of the system.

[4.0 Marks]

d) Investigate the stability of the equilibrium solutions.

[2.0 Marks]

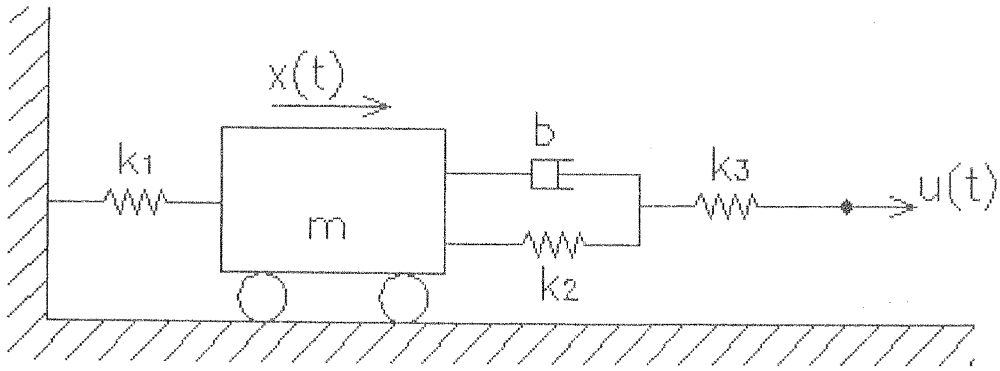

Figure Q2(b). Mechanical System

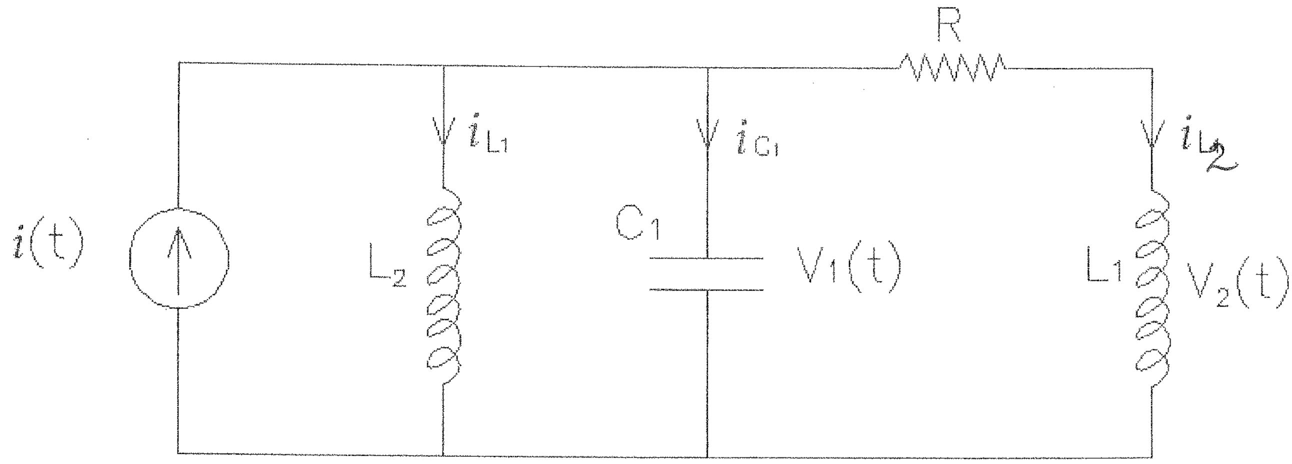

Figure Q4(a). RLC Circuit

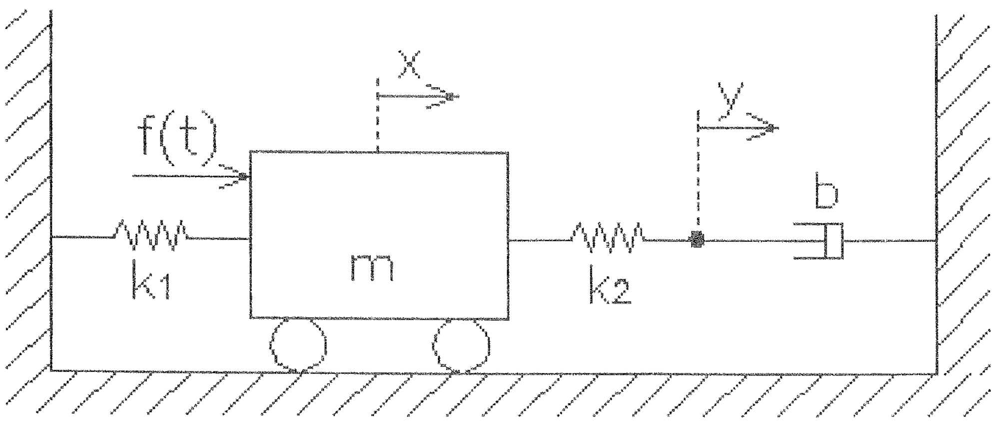

Figure Q4(b). Mass Spring Damper System

Table of Laplace transform pairs

$f(t)$	$F(s)$
$step$	$\frac{1}{s}$
$ramp, t$	$\frac{1}{s^2}$
$impulse$	1
$u(t-a)$	$\frac{e^{-as}}{s}$
$u(t-a)\ g(t-a)$	$e^{-as}\,G(s)$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{(s+a)}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{1}{(n-1)!}\,t^{n-1}\,e^{-at}$	$\frac{1}{(s+a)^n}$
$\sin\omega t$	$\frac{\omega}{(s^2+\omega^2)}$
$\cos\omega t$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin\omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos\omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$
$\frac{\omega_0}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_0 t}\,\sin(\omega_0\sqrt{1-\zeta^2 t})$	$\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$
$1-\frac{\omega_0}{\sqrt{1-\zeta^2}}\,e^{-\zeta\omega_0 t}\,\sin(\omega_0\sqrt{1-\zeta^2}\,t+\phi)$ $Where,\ \phi=\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$	$\frac{\omega_0^2}{s(s^2+2\zeta\omega_0 s+\omega_0^2)}$