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UNIVERSITY OF RUHUNA - FACULTY OF ALLIED HEALTH SCIENCES

DEPARTMENT OF PHARMACY

FIRST BPHARM PART I EXAMINATION - FEBRUARY 2022

PH1152 : MATHEMATICS - SEQ

TIME: TWO HOURS

INSTRUCTIONS

- There are **four** questions in this paper.
- Answer **all** questions.
- Calculators will be provided.
- No paper should be removed from the examination hall.
- Do not use any correction fluid.
- Use illustrations where necessary.

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1. a) Find the following limits:

(i) $\lim_{t \to 4} \frac{t^2 - 4t}{t^2 - 3t - 4}$ **[10]**

(ii) $\lim_{\ell \to 0} \frac{\sqrt{1+\ell} - 1}{\ell}$ **[15]**

b) The per capita growth rate r of *Escherichia coli* bacteria can be modelled by the function

$$r = \frac{sn}{k+n},$$

where n is the concentration of the nutrient, s is its saturation level, and k is a positive constant.

(i) Calculate $\frac{dr}{dn}$ using the **first principles**. **[25]**

(ii) Interpret $\frac{dr}{dn}$. **[10]**

(iii) Sketch r and $\frac{dr}{dn}$ on the same axes as n varies. **[25]**

c) In an experiment with the *protozoan Paramecium*, the protozoan population size $P(t)$ was modelled using the function

$$P(t) = \frac{61}{1 + 31e^{-0.7944t}},$$

where t is measured in days. According to this model, how fast was the population growing after 8 days? **[15]**

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2. a) Find an equation of the tangent line to the parabola $y = x^2 - 8x + 12$ at the point $(3, -3)$ and sketch both parabola and tangent line. [40]

b) Consider the function $f(x) = x^4 - 8x^2$.

(i) Find the stationary points of the function $f(x)$. [40]

(ii) Classify the above stationary points as maxima or minima using the second derivative $f''(x)$. [20]

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3. a) Consider the two variable function $f(x,y) = e^{x^2+y^2}$.

(i) Find the first partial derivatives $\dfrac{\partial f(x,y)}{\partial x}$ and $\dfrac{\partial f(x,y)}{\partial y}$. [10]

(ii) Verify that

$$\frac{\partial^2 f(x,y)}{\partial x \partial y} = \frac{\partial^2 f(x,y)}{\partial y \partial x}.$$

[20]

(iii) Find the total differential of f at the point $(1,1)$. [10]

b) Show that the function

$$g(x,y) = \frac{x^3 + y^3}{x - y}$$

is homogeneous of degree 2 and satisfies the Euler's theorem. [60]

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4. a) Using the method of integration by parts, show that

$$\int xe^x \, dx = xe^x - e^x + C,$$

where C is the constant of integration. [20]

Use the method of integration by parts and the above result to evaluate

$$\int_0^1 x^2 e^x \, dx.$$

[30]

b) If K is a constant, evaluate the integral

$$\int \frac{K}{N(K-N)} \, dN$$

using the method of partial fractions. [30]

c) Test the differential equation

$$(3x^2 + y\cos x)\, dx + (\sin x - 4y^3)\, dy = 0$$

for exactness. If it is exact, then find its solution. [20]
