Modeling the dynamics of fish and pest plant populations in a perennial tank

Mayuri HEP and Karunathilaka NGA

Department of Mathematics, Faculty of Science, University of Kelaniya, Sri Lanka

Corresponding author: mayurismail@gmail.com

The dynamic and qualitative behaviors of the populations of fish and pest have been modeled under the assumption that the pest is affected by beetles. The positivity and boundedness of the populations are established by considering the natural population of pests growing and the mortality associate to the infection as well as the loss of immunity of newborns. In the absence of time-delay the stability properties are investigated and numerical solution of the system of ordinary differential equations are simulated using the Runge-Kutta algorithm. Considering the dimensionless form of the system;

$$\frac{dx}{dt} = x(1-x) - \frac{axy}{b+x} - \frac{hxz}{n+x} - ex, \qquad x(0) > 0$$
$$\frac{dy}{dt} = \frac{gxy}{b+x} - sy - dyv, \qquad y(0) > 0$$
$$\frac{dz}{dt} = \frac{cxz}{n+x} - mz + dyv, \qquad z(0) > 0$$
$$\frac{dv}{dt} = \beta - \gamma v + fyv, \qquad v(0) > 0,$$

the following equilibrium points are obtained in the absence of time delay $\left(0,0,0,\frac{\beta}{\gamma}\right)$, $\left(1-e,0,0,\frac{\beta}{\gamma}\right)$, $\left(\frac{mn}{c-m},0,\frac{cn(c(1-e)-m(1-e+n))}{h(c-m)^2},\frac{\beta}{\gamma}\right)$, $\left(\frac{(s+dv)b}{g-s-dv},\frac{bg[(g-s-dv)(1-e)-(s+dv)b]}{a(g-s-dv)^2},0,v_3\right)$.

The pest free equilibrium point $\left(\frac{mn}{c-m},0,\frac{cn(c(1-e)-m(1-e+n))}{h(c-m)^2},\frac{\beta}{\gamma}\right)$ is further analyzed. The asymptotically stability condition $c\,(1-e) - m(1-e+n) > 0$ is derived. Here dimensionless quantities, a and h are the predation rates of the uninfected and infected pests respectively. b and n are the half saturation constants, c and g are the conversion factors. ex is the harvesting yield per unit time with e is a measure of the effort expended and s is the beetle-independent predator mortality rate, and d is the infection rate. β is the amount of beetle inserted in the field γ is the mortality rate of the beetle species (due to temperature changes, UV radiation etc.) And also assume that all parameters are positive and $\tau \in \mathbb{R}_+$. Comparison of the theoretical analysis and simulation results are compared, and it is concluded that if the fish population is attacked by high capacity and aggressive pest species, beetle infection pest become low capacity consumers.Then pests will be extinct and the fish population will survivethereby, the possibility of pest controlling is established.

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