UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: May 2022

Module Number: ME1202 Module Name: Introduction to Mechanical Engineering

[Three Hours]

[Answer **all questions**, each question carries **12** marks]

Clearly state all the assumptions that you may make.
To get full marks, make sure that you have answered with correct SI units and standard notations.
Take gravitational acceleration, **g = 9.8m/s^2**

Q1. a) The position of a particle that is having a curvilinear motion (Figure Q1(a)) is defined by a radial distance r from a fixed origin O and by an angular measurement θ to the radial line (measured from a fixed axis, x). The position vector of point A in space is described by $\boldsymbol{r} = \underline{r}(t)$. Take $\underline{e}_r$ and $\underline{e}_\theta$ are unit vectors along the r (radial) and θ (angular) directions.

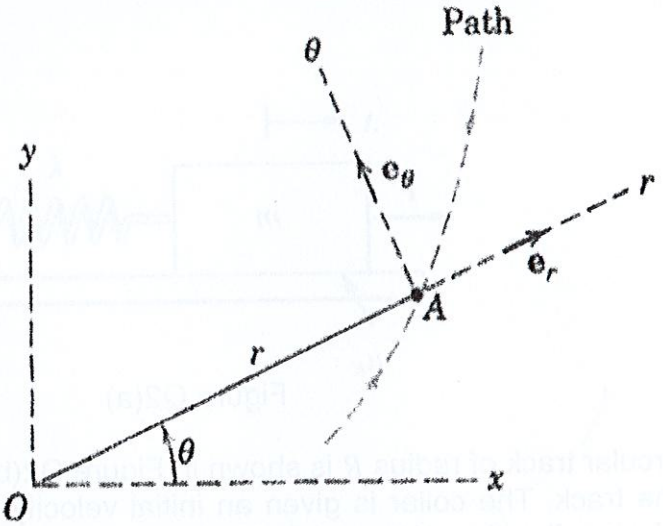


Figure Q1(a)

i) Write down the position vector of the particle.
ii) Obtain expressions for the first derivatives ($\dot{\underline{e}}_r$ and $\dot{\underline{e}}_\theta$) of unit vectors.
iii) Hence, derive expressions for the velocity vector ($\dot{\underline{r}}$) and the acceleration vector ($\ddot{\underline{r}}$) of the particle using above unit vectors.
iv) Determine the magnitude and the direction of the particle velocity and draw the directions of components and resultant velocity using the diagram (similar to Figure Q1(a)) given separately.
v) Determine the magnitude and the direction of the particle's acceleration and draw the directions of components and resultant ~~velocity~~ acceleration using the diagram (similar to Figure Q1(a)) given separately.

[7 Marks]

b) The polar coordinates of a position of a particle are given as $r = a(1 + \sin t)$ and $\theta = be^{-t}$ where t is in seconds and argument for the sine is in radians. r is measured in meters, θ is measured in radians, a and b are constants. When $t = 2\pi/3$ seconds, determine

i) the radial and transverse components of the particle's velocity,
ii) the radial and transverse components of the particle's acceleration and
iii) the direction of the total acceleration of the particle.

[5 Marks]

Q2 a) Figure Q2(a) shows a mass-spring system that consists of a block (mass, m) attached to a wall with a spring of constant k. The kinematic friction coefficient between the block surface and the horizontal plane is μ_k. A horizontal force, $F(t)$ is applied parallel to the plane. Taking the friction force on the cube as F_f and the displacement of the block as $x(t)$,

i) Draw free body diagrams showing the forces on the block when it is having a rectilinear motion in the same direction as of F and opposite to that (or when the block moves back and forth).
ii) Obtain F_f in vector form and show that $F_f = -\mu_k mg sign(\dot{x})$ where $sign(\dot{x}) = 1\ when\ \dot{x} > 0$ and $sign(\dot{x}) = -1\ when\ \dot{x} < 0$.
iii) Obtain the equation of the motion for the block.

[6 Marks]

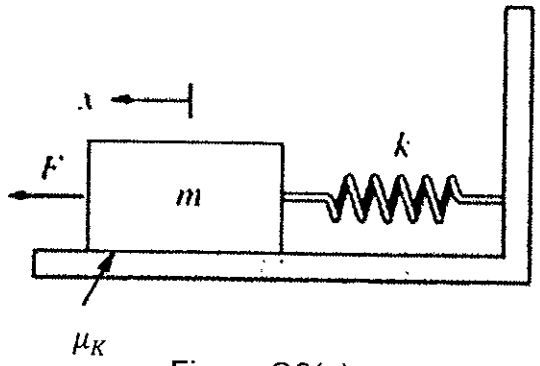


Figure Q2(a)

b) A circular track of radius R is shown in Figure Q2(b). A collar of mass m slides in the track. The collar is given an initial velocity v_0 in the tangential counter clockwise direction. If the coefficient of friction between the collar and the track is μ,

i) draw a free body diagram showing the normal reaction forces in vertical and radial directions, and the friction force.
ii) Determine the resultant normal force when the speed of the collar is v.
iii) Determine the acceleration of collar.
iv) Hence, show that $v\frac{dv}{ds} = -\frac{\mu\sqrt{R^2g^2+v^4}}{R}$.

[6 Marks]

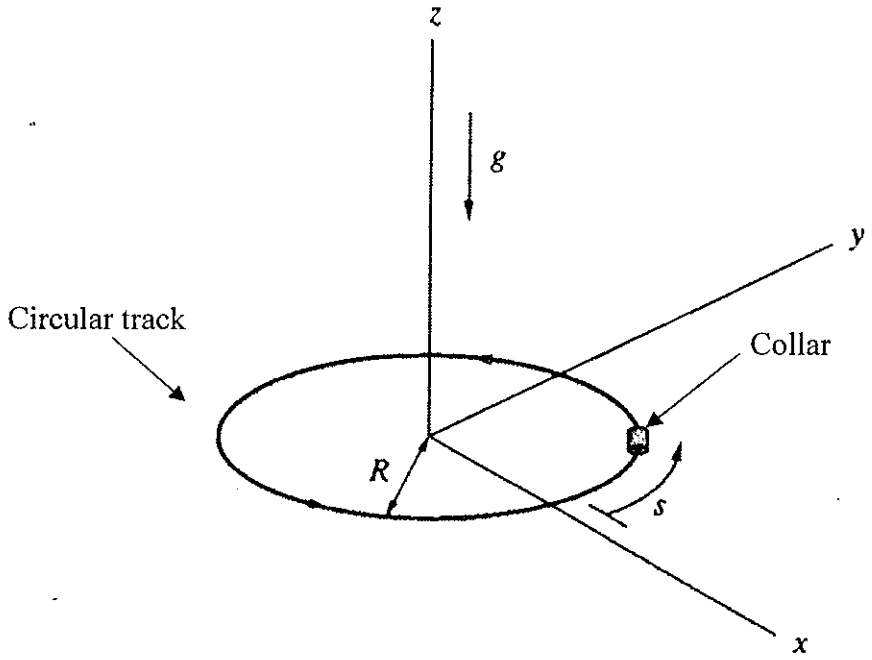


Figure Q2(b)

Q3 a) States the following theories used to analyse the dynamics of system of particles using appropriate mathematical relationships using standard notations. Define each term of the expressions.

i) Principle of linear impulse and momentum.

ii) Conservation of linear momentum.

iii) Conservation of energy.

[5 Marks]

b) Figure Q3 (b) shows a free-rolling 30° ramp of mass 40kg. A 10kg crate is released from rest at A and slides down 3.5 m to point B. Consider that the surface of the ramp is smooth and neglect the mass of the wheels attached to the ramp. Obtain a relationship between the ramp velocity and crate velocity when the crate reaches B using

i) the conservation of energy,

ii) the conservation of linear impulse and

iii) Determine the ramp velocity and crate velocity.

[7 Marks]

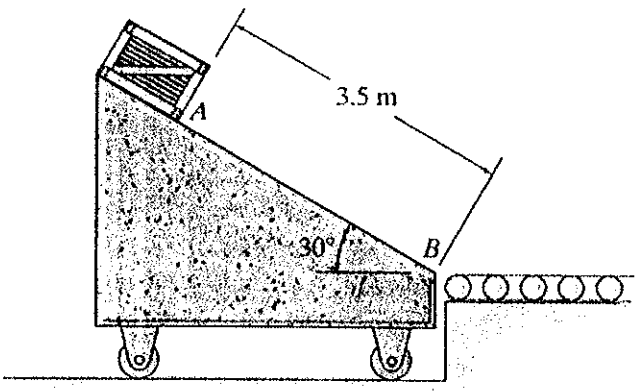


Figure Q3(b)

Q4 a) i) State the Newton's second law of motion for moments for system of particles.

ii) Hence, derive the principle of angular impulse and momentum for system of particles.

iii) "If no external impulse is applied to the particle, both linear and angular momentum will be conserved. In some cases, however, the particles angular momentum will be conserved but the linear momentum will not." When can this happen? Explain using above theories.

[6 Marks]

b) A disk brake mechanism is demonstrated in Figure Q4 (b). The 40kg disk is rotating at $V = 100\ \text{rad/s}$. The force P is applied to the brake as indicated by the graph. Taking the coefficient of kinetic friction at B is $\mu_K = 0.3$ and neglecting the thickness of the brake;

i) Draw the free body diagram showing all the forces on the disc.

ii) Determine the friction force by using force P.

iii) If the time t needed to stay the disk from rotating is greater than 2 seconds, calculate t.

[6 Marks]

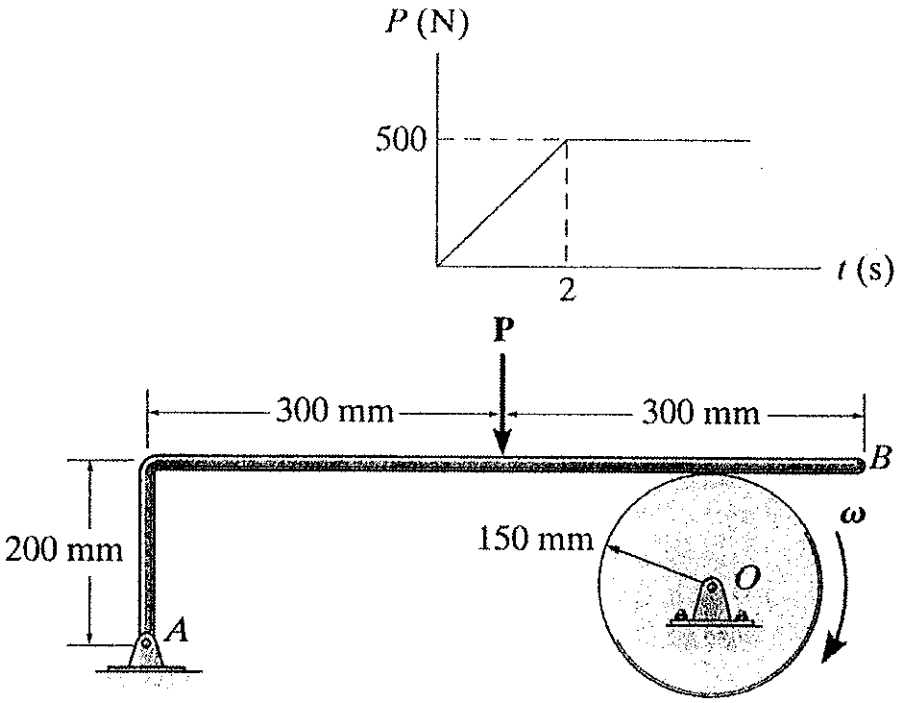


Figure Q4(b)

Q5 (a) Briefly describe 4 types of rigid body motion using suitable sketches

[4 Marks]

(b) A gear assembly shown in Figure Q5 (b) consists of a motor and two gears; B of pitch circle diameter $350mm$ and A of pitch circle diameter $200mm$. The motor coupled to the gear A gives an angular acceleration of $\alpha_A = a + b\theta^2\ rad/s^2$, where θ is in radians, a and b are constants. Take that the gear A is initially turning at $\omega_A\ rad/s$.

i) If the gears A and B are meshed as shown in the figure, obtain a relationship between the angular speeds of gear A and B.

ii) Determine the angular velocity of gear A after it undergoes an angular displacement of 10 revolutions.

iii) Using above (i) and (ii), determine the angular velocity of B.

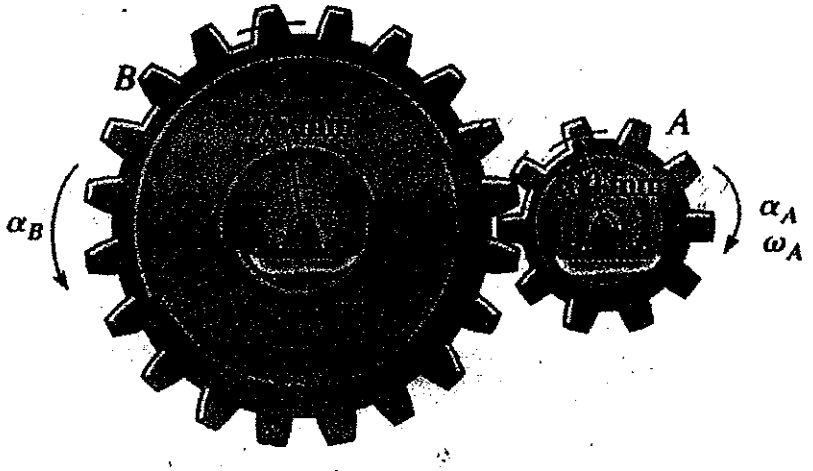


Figure Q5(b)

[5 Marks]

(c) Figure Q5(c) shows a kind of Newton's cradle with 5 steel balls of equal diameters and masses. All balls are attached to the top bar using similar strings and placed at equal space. You may remember this setup was demonstrated during one of the lectures. When you pull one end ball away (say right), the left most ball travels the same distance as the right ball. It will then swing back and hit the stationary balls, where the cycle is repeated.
Briefly explain the principle behind this motion and why the motion cannot be maintained forever.

[3 Marks]

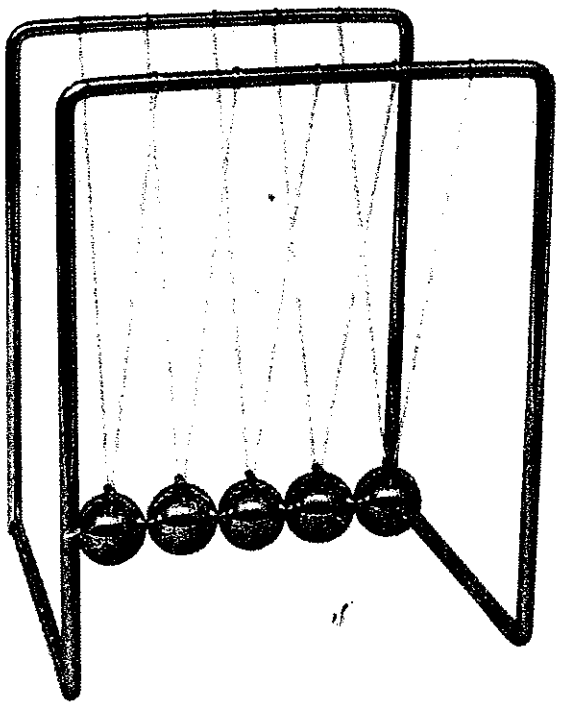
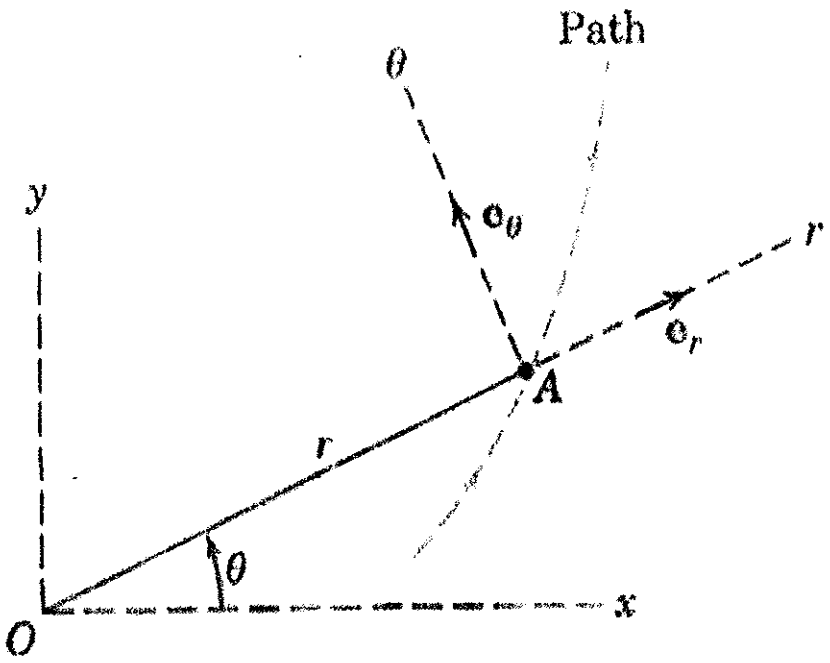
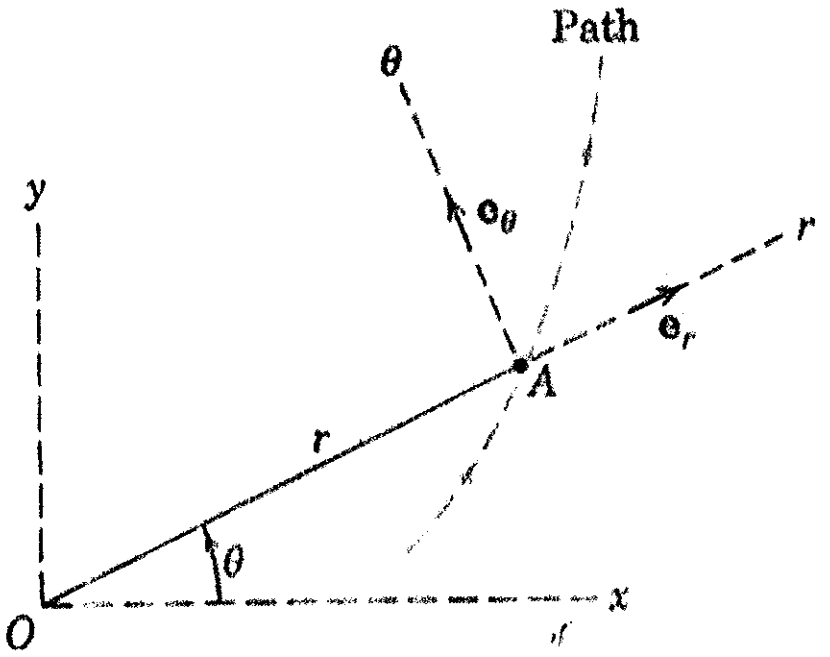


Figure Q5(c)

This page is for Q1(a). Attach this to your answer script.



Q1(a)(iv)



Q1(a)(v)