Chin. Phys. B Vol. 27, No. 7 (2018) 070306

RAPID COMMUNICATION

Momentum distribution and non-local high order correlation functions of 1D strongly interacting Bose gas*

EJKP Nandani[1,2,3] and Xi-Wen Guan(管习文)[1,4,†]

[1] Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan 430071, China
[2] University of Chinese Academy of Sciences, Beijing 100049, China
[3] Department of Mathematics, University of Ruhuna, Matara 81000, Sri Lanka
[4] Department of Theoretical Physics, Research School of Physics and Engineering, Australian National University, Canberra ACT 0200, Australia

(Received 2 May 2018; revised manuscript received 16 May 2018; published online 25 June 2018)

The Lieb–Liniger model is a prototypical integrable model and has been turned into the benchmark physics in theoretical and numerical investigations of low-dimensional quantum systems. In this note, we present various methods for calculating local and nonlocal M-particle correlation functions, momentum distribution, and static structure factor. In particular, using the Bethe ansatz wave function of the strong coupling Lieb–Liniger model, we analytically calculate the two-point correlation function, the large moment tail of the momentum distribution, and the static structure factor of the model in terms of the fractional statistical parameter $\alpha = 1 - 2/\gamma$, where γ is the dimensionless interaction strength. We also discuss the Tan's adiabatic relation and other universal relations for the strongly repulsive Lieb–Liniger model in terms of the fractional statistical parameter.

Keywords: correlation function, momentum distributions, structure factor, contact

PACS: 03.75.Ss, 03.75.Hh, 02.30.IK, 05.70.Ce **DOI:** 10.1088/1674-1056/27/7/070306

1. Introduction

The Bethe ansatz, which was introduced in 1931 by Hans Bethe, has become a powerful method to obtain exact solutions of one-dimensional (1D) quantum many-body systems. In 1963, Lieb and Liniger[1] solved the 1D many-particle problem of δ-function interacting bosons by the Bethe's hypothesis. The ground state, the momentum, and the elementary excitations were obtained for this model by using the Lieb–Liniger solution. In this context, a significant step was made on the discovery of the grand canonical description of this Lieb–Liniger model by Yang and Yang in 1969.[2] Now, this grand canonical approach is called Yang–Yang thermodynamic method. The Yang–Yang thermodynamics of the Lieb–Liniger Bose gas provides benchmark understanding of quantum statistics, thermodynamics, and quantum critical phenomena in many-body physics, see a review.[3,4] In the context of ultracold atoms, the 1D Bose gas with a repulsive short-range interaction characterized by a tunable coupling constant exhibits rich many-body properties. This model thus becomes an ideal test ground to explore fundamental many-body phenomena ranging from equilibrium to nonequilibrium physics in the experiment.[5–10]

Despite Lieb–Liniger is arguably the simplest integrable model, the calculation of the correlation functions is extremely challenging due to the complexity of the Bethe ansatz many-body wave function of the model. The study of correlation functions has long been being an important theme in the physics of ultracold quantum gases since they provide information of quantum many-particle interference and coherence beyond the solely spectra of the systems. Therefore, there has been much theoretical and experimental interest in the local, non-local, and dynamical correlation functions at zero and finite temperatures via numerous methods based on exactly solvable models, see Refs. [11]–[22].

For sufficiently strong interaction and sufficiently low density, the 1D Lieb–Liniger gas enters the Tonks–Girardeau (TG) regime, in which bosons behave like impenetrable particles (hard-core bosons). Such impenetrable bosons behave mostly like the free fermions that build up the Girardeau's Bose–Fermi mapping.[23] In fact, the 1D Lieb–Liniger Bose gas with the interacting strength $c_{\rm B}$ can map onto the fully-polarized fermions with a p-wave interaction of strength $c_{\rm F} = 1/c_{\rm B}$.[24] As a result of the Bose–Fermi mapping, the energy spectra of the Bose and corresponding Fermi systems are identical at the TG regime. The observables that can be given in terms of the local density are identical for both systems, such as dynamical density–density correlation function, see Ref. [25]. However, this mapping for the off-diagonal correlation function does not like to be true, for example, the momentum distribution. The Bose–Fermi mapping has tremen-

*Project supported by the National Natural Science Foundation of China (Grant Nos. 11374331 and 11534014) and the National Key R&D Program of China (Grant No. 2017YFA0304500). This work has been partially supported by CAS-TWAS President's Fellowship for International PhD Students.

†Corresponding author. E-mail: xwe105@wipm.ac.cn

 http://iopscience.iop.org/cpb http://cpb.iphy.ac.cn