UNIVERSITY OF RUHUNA

FACULTY OF FISHERIES AND MARINE SCIENCES & TECHNOLOGY

Academic Year 2023/2024

Bachelor of Science Honours in Marine and Freshwater Sciences Degree
Level I Semester I Examination – November/December 2024

OCG 1141: Mathematics **Time: 1 hour and 30 minutes**

Answer any **three (03)** questions.

Q1

(a) Solve the equation

$$\log_3 x + \log_3(x-2) = 2.$$

for x. [25 marks]

(b) A nutritionist wants to create a diet plan that provides at least 3000 calories but does not exceed 4500 calories per day. The diet consists of two foods: A and B. Let x be the number of servings of food A, and y be the number of servings of food B. The calories per serving are given as:

Food A: 200 calories
Food B: 150 calories

(i) Formulate a system of inequalities representing the caloric constraints. [05 marks]

(ii) Determine if the combination $x = 10$ and $y = 15$ satisfies the constraints. [10 marks]

(c) Find the domain, co-domain and range of the function $f(x) = \frac{2x+3}{x-1}$. [20 marks]

(d) Consider the function $f(x) = 3x + 5$.

(i) Find the inverse of the function, $f^{-1}(x)$. [05 marks]

(ii) Verify that $f\left(f^{-1}(x)\right) = x$. [10 marks]

(e) Use the Cramer's rule and solve the system of equations

$$3x + 4y = 10$$
$$5x - 2y = 8$$

for x. [25 marks]

Q2

(a)

(i) Write the formulas for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$. [10 marks]

(ii) Hence prove that $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$. [15 marks]

(iii) Using the above formulas find $\sin 75^0$, $\cos 75^0$ and $\tan 75^0$. [25 marks]

(iv) Take $\beta = 2\alpha$ and using the formulas in part (i), show that [15 marks]

$$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha.$$

(b) Verify the following identities.

(i) $\frac{\sin x}{\sin 2x} + \frac{\cos x}{\cos 2x} = 2 \sin 3x \ \operatorname{cosec} 4x.$ [20 marks]

(ii) $\frac{\cos 2x - \cos 12x}{\sin 12x + \sin 2x} = \tan 5x.$ [15 marks]

Q3

(a) Find the following limits.

(i) $\lim_{x \to 2} \frac{\sin(x-2)}{x^2 - 4}$ [20 marks]

(ii) $\lim_{x \to \frac{1}{2}} \frac{2x^2 - 3x + 1}{6x^2 - 5x + 1}$ [15 marks]

(b) The concentration of a drug in the bloodstream $C(t)$ (in mg/L) after t hours is given by:

$$C(t) = 20\, t\, e^{-0.5t}.$$

(i) Find $\frac{dC(t)}{dt}$. [10 marks]

(ii) Setting $\frac{dC(t)}{dt} = 0$, find the time t at which the concentration is maximized. [15 marks]

(iii) Hence, determine the maximum concentration. (Assume $e = 2.72$) [10 marks]

(c) If $y = \frac{1}{x} \sin x$, show that [30 marks]

$$x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + yx = 0.$$

Q4

(a) Using the method of integration by parts, evaluate the definite integral [25 marks]

$$\int_0^2 x e^{x^2}\, dx.$$

(b) Find the constants A, B and C such that,

$$\frac{2x+5}{x\,(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

Hence evaluate $\int \frac{2x+5}{x\,(x^2+1)}\, dx$. [35 marks]

(c) A population of animals, under certain conditions, has a growth rate given by $500\pi \cos 2\pi t$, where t is the time in years. If the initial size of the herd is 3000, find the size of the population at time t. What are the maximum and minimum numbers in the herd during the course of a year? [40 marks]

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