UNIVERSITY OF RUHUNA

FACULTY OF FISHERIES AND MARINE SCIENCES & TECHNOLOGY

Academic Year 2023/2024

Bachelor of Science Honours in Marine and Freshwater Sciences Degree
Level III Semester I Examination – Oct/Nov 2024

OCG 3172: Mathematics II **Time: 02 (Two) hours**

Answer **any four (04)** questions. Each question carries equal marks.

Q1

(a) Consider the function $f(x,y) = \begin{cases} \frac{x^2-y^2}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$

Show that $f(x,y)$ is not continuous at the point $(0,0)$. [25 marks]

(b) If $v = (x^2+y^2+z^2)^{\frac{m}{2}}$, find the value of m $(m \neq 0)$ such that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0 .$$ [40 marks]

(c) (i) State Euler's theorem for a function of two variables. [10 marks]

(ii) Verify Euler's theorem for $f(x,y) = \frac{x^3}{y} + y^2$. [25 marks]

Q2

(a) Explain a scalar and a vector giving two examples for each.

[30 marks]

(b) Let $\underline{a}$ and $\underline{b}$ be any two non-zero vectors in $\mathbb{R}^3$. Define, in the usual notation,

(i) the scalar product $\underline{a} \cdot \underline{b}$.

(ii) the vector product $\underline{a} \times \underline{b}$. [20 marks]

(c)

(i) Consider the three vectors $\underline{a} = -4\underline{i} + 2\underline{j}$, $\underline{b} = 2\underline{i} + \underline{j}$ and $\underline{c} = 2\underline{i} + 3\underline{j}$. If $\underline{c} = m\,\underline{a} + n\,\underline{b}$, find the values of m and n. [10 marks]

(ii) If $\underline{p}$ and $\underline{q}$ are two vectors where $\underline{p} = 2\underline{i} + 5\underline{j}$ and $\underline{q} = \alpha\underline{i} + 4\underline{j}$, find α such that $\underline{p}$ is perpendicular to $\underline{q}$. [10 marks]

(iii) The straight line l_1 passes through the points $A = (2,5,9)$ and $B = (6,0,10)$. Find the vector equation of l_1.

The straight line l_2 has vector equation $\underline{r} = \begin{pmatrix} 8 \\ 8 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$, where μ is a real parameter. Show that the point A is the intersection of l_1 and l_2.

Further, show that l_1 and l_2 are perpendicular to each other. [30 marks]

Q3

(a) Let $\underline{a}, \underline{b}$ and $\underline{c}$ be any three non-zero vectors in $\mathbb{R}^3$. Define, in the usual notation,

(i) the scalar triple product,

(ii) the vector triple product. [10 marks]

(b) Find the volume of a parallelepiped formed by three coterminous edges represented by vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ given by $\underline{i} - \underline{j} + \underline{k}$, $2\underline{i} + 3\underline{j} - \underline{k}$, and $-\underline{i} - \underline{j} + 5\,\underline{k}$, respectively.

[20 marks]

(c) A particle moves along the curve $\vec{r} = (t^3 - 3t^2 + 2\ ,\ 2t^3 + t,\ 4t^3 - 1)$, where t is time. Find the velocity and acceleration when $t = 3$. [30 marks]

(d) Consider the function $f(t) = (t, t^2, t^3)$ and $g(t) = (e^t, \sin t\ , \ln t)$. Find the derivative $\frac{d}{dt}\,[f(t)\ .\ g(t)]$, where $f(t).g(t)$ is the scalar product of f and g. [40 marks]

Q4

(a) Use reduced row echelon form to solve the system of linear equations given by

$$x_1 + x_2 + 2x_3 = 3$$
$$2x_1 + 2x_2 + 4x_3 = 6$$
$$x_1 + x_2 + x_3 = 2.$$

[35 marks]

(b) Let $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} ; x_1 , x_2 \in \mathbb{R} \right\}$ be a set with addition and scalar multiplication defined by $v_1 \oplus v_2 = \begin{bmatrix} x_1 - 2x_2 \\ 2x_1 + x_2 \\ 0 \end{bmatrix}$ and $c \otimes v_1 = \begin{bmatrix} cx_1 \\ c^2 x_2 \\ c \end{bmatrix}$ respectively, where c is a scalar. Check whether V is a vector space or not. [30 marks]

(c) Determine whether the vectors

$v_1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$, $v_3 = \begin{bmatrix} -2 \\ -5 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$ are linearly dependent or independent.

Provide your reasoning and detailed calculations clearly. [35 marks]

Q5

(a) Define the following in the usual notation.

(i) Gradient of a scalar function $\emptyset(x, y, z)$,

(ii) Divergence of a vector function $\vec{\boldsymbol{F}}$,

(iii) Curl of a vector function $\vec{\boldsymbol{F}}$. [30 marks]

(b) Find the divergence and the curl of $\vec{F} = x^2y\,\underline{i} + y^2z\,\underline{j} + z^2x\,\underline{k}$ at $(1, -1, 1)$. [40 marks]

(c) Express the point $\left(3, \frac{2\pi}{3}, 6\right)$ given by cylindrical coordinates in both cartesian and spherical coordinates. [30 marks]

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