UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 1 Examination in Engineering: March, 2026

Module Number: IS1402 **Module Name: Mathematical Fundamentals for Engineers (C – 23)**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1. a) Let $z = \sqrt{3} - i$.

i.) Convert z into polar form.

ii.) Write down the modulus and the argument of z .

[2 Marks]

b) Find the complex number $w = a + ib,$ such that $w^2 = z$, where $a, b \in \mathbb{R}$ and

$$z = 5 - 12i.$$

[2 Marks]

c) i.) State the De Moivre's Theorem for fractional powers of the type $^1/_p$, where $p \in \mathbb{N}$.

ii.) Use De Moivre's theorem to find the cubic roots of $8i$ and and represent the roots graphically.

iii.) For $n \in \mathbb{N}$, express $\cos^n \theta$ in terms of multiples of cosines and hence, evaluate

$$\int_0^{\pi} \cos^4 \theta \sin^2 \theta d\theta.$$

[8 Marks]

Q2. a) Briefly explain the following types of matrices by giving a non-trivial example for each.

i.) Diagonal matrix

ii.) Hermition matrix

iii.) Orthogonal matrix

iv.) Idempotent matrix

[3 Marks]

b) i.) Show that all the diagonal elements of a Hermitions matrix are real.

ii.) Show that any square matrix A can be written as the sum of a symmetric and a skew-symmetric matrices.

iii.) Let A and B be non-singular square matrices. Show that,

$$(AB)^{-1} = B^{-1}A^{-1}.$$

[3 Marks]

c) i.) Find the values of α and β such that the following system has

unique solution

infinitely many solutions or

no solutions

$$\begin{aligned} 2x + \alpha y - z &= 8 \\ x - y - 3z &= 5 \\ x + 3y + 2z &= \beta \end{aligned}$$

ii.) When $\alpha = 1$ and $\beta = 4$ solve the above system in i.) by using Cramer's rule.

[6 Marks]

Q3. a) Let $f: \mathbb{R} \to \mathbb{R}$ be a real valued function and $a \in \mathbb{R}$. Explain what is meant by

i.) f is a one to one.

ii.) f is continuous at a.

iii.) f is differentiable at a.

[3 Marks]

b) i.) Let f and g be one to one functions. Show that $f \circ g$ is also a one to one function.

ii.) Show that if f is finitely differentiable at $a \in \mathbb{R}$ then f is continuous at a.

iii.) Find the left handed and right handed limits at $x = 0$ of the function

$$f(x) = \frac{e^{\frac{1}{x}} - 2}{e^{\frac{1}{2x}} + 1}; \text{ where } x \neq 0.$$

Hence, determine whether the limit of $f(x)$ when x tends to zero exist or not.

[5 Marks]

c) Evaluate the following limits.

i.) $\lim_{x\to 0} \left(\frac{1 - \cos^3 x}{\sin^2 x} \right)$ ii.) $\lim_{x\to\infty} \left(\frac{1 - e^x}{1 + e^x} \right)$

[2 Marks]

d) Sketch the graph of $y = 2|x + 1| + |x - 3| - 3|x - 1|$.

[2 Marks]

Q4. a) i.) State the Rolle's Theorem

ii.) If f and g are continous on $[a,b]$ and differentiable on (a,b) with $g'(x) \neq 0$ for all $x \in (a,b)$, prove that there exist $c \in (a,b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b)-f(a)}{g(b)-g(a)}.$$

iii.) State the Mean Value Theorem.

iv.) Use Mean Value Theorem to show, for $x > 0$

$$\ln(1+x) < x < e^x - 1.$$

[6 Marks]

b) i.) State and prove the Euler's theorem on homogeneous functions.

ii.) Use Euler's theorem to show that if

$$u = \cos^{-1}\left(\frac{x^4-y^4}{x^2-2xy+3y^2}\right),$$

then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + 2\cot u = 0.$$

iii.) If $z = f(x,y)$, where $x = e^r\cos\theta$ and $y = e^r\sin\theta$, show that

$$\left(\frac{\partial z}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial \theta}\right)^2 = e^{2r}\left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right].$$

[6 Marks]

Q5. a) i.) Briefly explain what is meant by 'position vector' and 'unit vector'.

ii.) Let $\mathbf{a} = \mathbf{i} + \lambda\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = \mathbf{i} - 3\mathbf{j} + \lambda\mathbf{k}$ be position vectors of the points A and B respectively, where λ is a constant. If O is the orign then determine the value of λ such that OAB is a rectangular triangle with $\angle OAB = \pi/2$.

iii.) Let the position vectors of three points A, B and C are, $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$, $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, and $\mathbf{c} = 3\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ respectively. Show that **a**, **b**, and **c** are not coplanar and hence, determine the area of the parrellepipped formed by these three vectors.

iv.) Find the vector triple product of **a**, **b** and **c**, in above iii.)

[6 Marks]

b) i.) Let $\phi(x,y,z) = x^2z^2 - yz$. Find a unit vector perpendicular to the surface $\phi(x,y,z) = 4$ at the point $(1,1,-1)$.

ii.) Find the directional derivative of ϕ given in ~~ii~~ i.) above at the point $(1,1,-1)$ in the direction of $\mathbf{a} = 2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$,

iii.) If the position vector of any arbitrary point (x,y,z) is denoted by $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ and $|\mathbf{r}| = r$, show that

$$\nabla.\left(\frac{\mathbf{r}}{r^2}\right) = \frac{1}{r^2}$$

[6 Marks]