# UNIVERSITY OF RUHUNA

## Faculty of Engineering

End-Semester 8 Examination in Engineering: December 2025

**Module Number: EE8204** **Module Name: Digital Communication**

**[Three Hours]**

[Answer **all questions,** each question carries **25 marks]**

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**Notations and symbols have their usual meaning unless otherwise stated.**
**If necessary, if you have any doubt as to the interpretation of the wording of a question make your own decision, and clearly state it.**

Q1 a) Let the two signals be defined as follows:

$$s_1(t) = A \cdot p(t), \quad s_2(t) = A \cdot p(t - T)$$

where $p(t)$ is a rectangular pulse defined by

$$p(t) = \begin{cases} 1 & 0 \leq t < T \\ 0 & Otherwise. \end{cases}$$

The orthonormal basis functions for the signal space are:

$$\phi_1(t) = \frac{1}{\sqrt{T}}p(t), \quad \phi_2(t) = \frac{1}{\sqrt{T}}p(t - T)$$

i) Express $s_1(t)$ and $s_2(t)$ as vectors in the signal space defined by $\phi_1(t)$ and $\phi_2(t)$. [4.0 Marks]

ii) Compute the Eucledian distance between $s_1(t)$ and $s_2(t)$. [2.0 Marks]

iii) Sketch $s_1(t)$ and $s_2(t)$ in a signal space diagram. [4.0 Marks]

b) Consider a binary signal set $S = \{s_0(t), s_1(t)\}$, where:

$$s_0(t) = A \cdot q(t), \quad s_1(t) = A \cdot q(t) + B \cdot q(t - T).$$

Here $q(t)$ is a unit-amplitude rectangular pulse of duration $T$, and $A, B > 0$ are constants.

i) Determine the average energy of the signal set $S$ assuming both signals are equally likely. [3.0 Marks]

ii) Design a new signal set $S_{\min} = \{s_0'(t), s_1'(t)\}$ that maintains the same Euclidean distance between the two signals but minimizes the average energy. Compute the new average energy. [5.0 Marks]

c) The binary orthogonal signal set, $s_k = \sqrt{E}e^{jk\pi/2}$, $k = 0,1$, where $E$ is the energy of each signal, is used to transmit equi-probable binary signals over an additive white Gaussian noise (AWGN) channel with noise power spectral density $\frac{N_0}{2}$ per dimension. Assuming equi-probable signals and maximum-likelihood decoding, find the bit error probability of a binary phase shift keying (BPSK) signal set having the same average energy as the binary orthogonal signal set. Compare the bit error probability of the two signal sets.

[7.0 Marks]

Q2 a) Consider the signal $s(t)$ being transmitted over an AWGN channel with noise power spectral density $\frac{N_0}{2}$ per dimension, where

$$s(t) = \begin{cases} 1 - \frac{t}{T}, & 0 \leq t \leq T \\ 0, & \text{otherwise}. \end{cases}$$

i) Sketch the impulse response of a matched-filter receiver for this signal, highlighting the impulse response and the sampling instance of the receiver.

[4.0 Marks]

ii) Determine the signal power at the sampling instance.

[4.0 Marks]

iii) Determine the signal-to-noise ratio at the sampling instance.

[3.0 Marks]

b) Suppose you are tasked with designing a digital communication system using an M-ary modulation scheme whose average symbol error probability is given by

$$P_s = 4Q\left(\sqrt{\frac{3\,SNR}{\frac{31}{32}M - 1}}\right),$$

where $M$ (an integer power of 2) is the size of the signal set, $Q(.)$ is the Gaussian Q-function, and $SNR$ is the signal-to-noise ratio. The system must transmit $10^6$ symbols per second while maintaining the symbol error probability (SEP) below $2 \times 10^{-6}$.

i) Calculate the maximum achievable bit rate with this system if $SNR = 30.2$ dB. (Hint: You may use inverse Q function value: $Q^{-1}(0.5 \times 10^{-6}) = 4.8916$.)

[5.0 Marks]

ii) Suppose due to poor channel conditions, the $SNR$ is now dropped to 18 dB. If the system is required to maintain the same SEP, state the modification it requires and calculate the achievable bit rate with the suggested modification.

[5.0 Marks]

c) A certain set of signals is constructed by sampling the unit circle into $2n$ equal segments along the circumference, with $n(> 1) \in Z^+$.

i) If we are to represent each signal using a fixed length binary encoding technique, determine the required number of bits.

[2.0 Marks]

ii) Find the minimum Euclidean distance between any two signals.

[2.0 Marks]

Q3 a) Pulse shaping is an important step in digital data transmission.

i) Briefly explain why a stream of rectangular pulses causes inter-symbol interefence in a band-limited channel.

[3.0 Marks]

ii) A pulse $p(t)$, whose spectrum is shown in Figure Q3 a), satisfies Nyquist's criterion. If $f_1 = 0.8$ MHz and $f_2 = 1.2$ MHz, determine the maximum rate at which binary data can be transmitted by using this pulse.

[3.0 Marks]

iii) Determine the roll-off factor of the pulse in part Q3 a) ii).

[2.0 Marks]

iv) Suppose that the frequencies $f_1$ and $f_2$ are adjustable. Determine the values of $f_1$ and $f_2$ such that this pulse can be used to transmit binary data at a rate of 1 Mbit/s over a baseband channel of bandwidth 700 kHz.

[4.0 Marks]

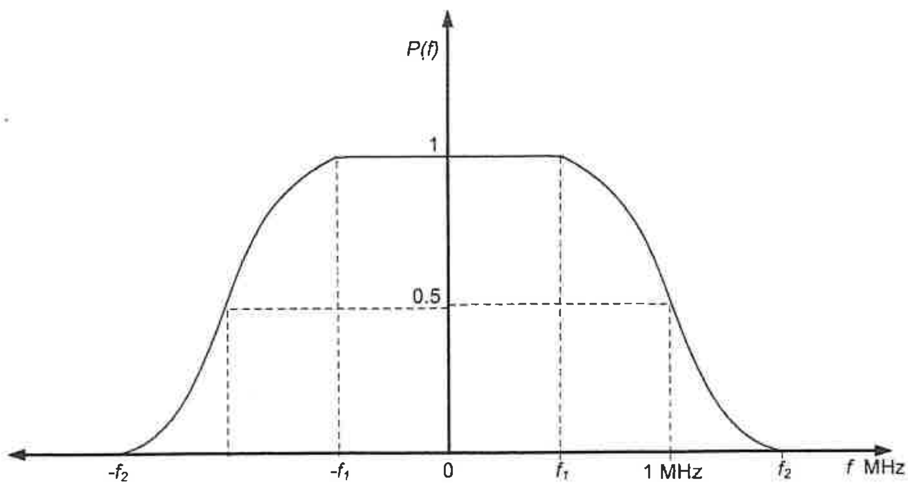


Figure Q3 a) - Nyquist pulse

b) A communication channel passes the frequencies in the band 600 to 3600 Hz. Suppose you are designing a modem to achieve a data rate of 9600 bits/s.

i) What is the theoretical maximum signaling (symbol) rate achievable through this channel using a real modulation scheme, satisfying Nyquist's criteria for inter-symbol interference free transmission?

[3.0 Marks]

ii) With proper justifications, select a real modulation scheme for your design.

[4.0 Marks]

iii) Determine the roll-off factor of a raised-cosine family pulse, which utilizes the entire available bandwidth with the modulation scheme suggested in part Q3 b) ii).

[4.0 Marks]

iv) Suggest a suitable carrier frequency for your design.

[2.0 Marks]

Q4 a) A received pulse of a baseband digital data transmission through a band-limited channel is shown in Figure Q4 a). You are supposed to design a 3-tap zero-forcing equalizer to obtain a Nyquist criteria pulse.

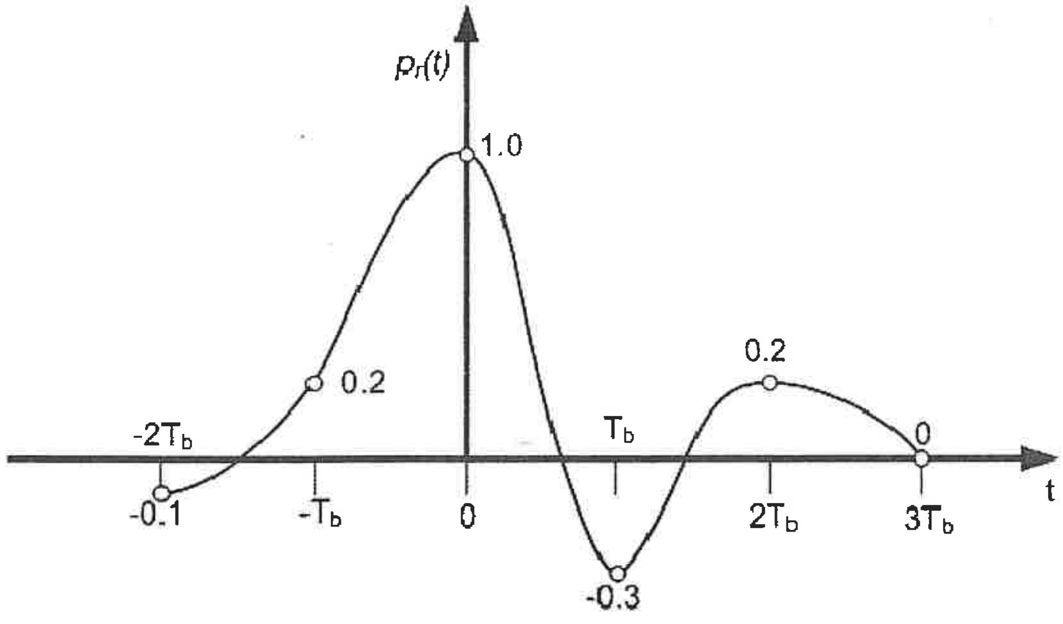


Figure Q4 a)

i) Determine the input-output relationship of the equalizer in matrix form.

[4.0 Marks]

ii) Obtain the tap setting of the equalizer.

[4.0 Marks]

b) The 8-ary signal set

$$s_k = e^{j\frac{\pi}{2}k}, \quad and \quad w_k = \sqrt{\alpha}e^{j\frac{\pi}{4}(2k+1)},$$

where $k = 0, 1, 2, 3$, and $0 < \alpha \leq \frac{1}{2}$ is a constant, is used to transmit 8 equi-probable signals over an AWGN channel, where the noise power spectral density is $\frac{N_0}{2}$ per dimension.

i) Classify the signal set into one of the known constellations (i.e., Pulse Amplitude Modulation (PAM), Phase Shift Keying (PSK), Quadrature Amplitude Modulation (QAM).

[2.0 Marks]

ii) What is the advantage of using the above complex constellation instead of a real constellation with the same number of signal points?

[3.0 Marks]

iii) Find the average energy of the signal set.

[3.0 Marks]

iv) Assuming maximum likelihood decoding, clearly draw the optimal decision regions for $0 < \alpha \leq 1/2$.

[4.0 Marks]

v) Let $P(C|s_k)$ denote the probability of making a correct decision given $s_k$ is transmitted. Obtain an expression for $P(C|s_k)$ when $\alpha = 1/2$.

[5.0 Marks]