University of Ruhuna

Bachelor of Science General Degree

Level I (Semester I) Examination

June 2023

Subject: Mathematics
Course Unit: MAT1142 (Mathematics for Biology)
Time: Two (02) Hours

Answer **ALL** questions. Calculators will be provided.

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1. a) Consider the complex number $z = 2 + i\sqrt{2}$, where i is the imaginary unit.

(i) Write down the complex conjugate $\bar{z}$ of z.

(ii) show that $z + \bar{z}$ and $z\bar{z}$ are real numbers.

(iii) calculate the modules of z. i.e. $|z|$.

[20 marks]

b) Find a and b such that $a + ib = \dfrac{z_1 z_2}{z_1 - z_2}$, where $z_1 = 2 + 3i, z_2 = 1 - 2i$ and i is the imaginary unit. [20 marks]

c) Using the Binormial expansion, show that

$$\left(t + \frac{2}{t}\right)^5 = t^5 + 10t^3 + 40t + \frac{80}{t} + \frac{80}{t^3} + \frac{32}{t^5}.$$ [20 marks]

d) Solve the equation :

$$\log_8 x + \log_8(x + 6) = \log_8(5x + 12).$$ [20 marks]

e) Verify the trigonometric identity

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin 2\theta}.$$ [20 marks]

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2. a) Find the following limits:

(i) $\displaystyle\lim_{x \to 2} \frac{x^2 + 4x - 12}{x^2 - 2x}$,

(ii) $\displaystyle\lim_{x \to \infty} \frac{5x}{x + 1}$. [15 marks]

b) Find the first derivative of the following functions with respect to x:

(i) $y = \sqrt{x}\sin x + x^4$,

(ii) $y = (x^2 - 5)^4$,

(iii) $y = e^{3x}\ln x$,

(iv) $y = \dfrac{x^2 + x + 1}{x - 1}$, $x \neq 1$.

[45 marks]

c) The spread of a disease among certain population can be modeled by $P(t) = \frac{20t^3 - t^4}{1000}$, $0 \leq t \leq 20$, where P is the number of people infected (in hundreds) and t is the time (in weeks).

(i) Find $P(t)$ after 5 weeks.

(ii) When the population with disease is maximum?
Use the **second criteria of derivatives** to justify your answer.

(iii) Find the maximum value of $P(t)$.

[40 marks]

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3. a) Let $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ be the relationship between pressure (P), volume (V) and temperature (T) of a gas. a, b and R are constants.

(i) Show that $P = \frac{RT}{(V-b)} - \frac{a}{V^2}$.

(ii) Find $\left(\frac{\partial P}{\partial T}\right)_V$ and $\left(\frac{\partial P}{\partial V}\right)_T$.

(iii) Show that

$$\left[\frac{\partial}{\partial V}\left(\frac{\partial P}{\partial T}\right)_V\right]_T = \left[\frac{\partial}{\partial T}\left(\frac{\partial P}{\partial V}\right)_T\right]_V$$

(iv) Write down the expression for the total differential of P in the usual manner.

[35 marks]

b) In a continuously assessed course unit, the marks on three different parts carry $20\%, 35\%$ and 45% of the total marks. A student record $65\%, 45\%$ and 55% on these parts respectively. Calculate the final mark of the student.

[10 marks]

c) The following table contains the percentage marks scored by the students $S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}$ and S_{11} respectively.

Student	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}
Marks Scored	x	$x+10$	60	50	40	60	50	80	$x-10$	50	60

If the mean of the students marks is 60, find x.
Find the mode, median, range, mean deviation, sample variance and the standard deviation of the marks scored by the students. **[55 marks]**

Continued.

4. a) Use the method of integration by parts to evaluate $\int x \ln x \, dx$.

[25+10 marks]

b) Use the result
$$\frac{3x^2+x+1}{x+1} = (3x-2) + \frac{3}{x+1}$$
to evaluate $\displaystyle\int \frac{3x^2+x+1}{x+1}\, dx.$ [15 marks]

c) Show that $\displaystyle\int_2^3 \frac{3x}{x^2+5}\, dx = \frac{3}{2} \ln \frac{14}{9}.$ [15 marks]

d) Find the constants A and B such that
$$\frac{x+7}{x^2-x-6} = \frac{A}{x-3} + \frac{B}{x+2}.$$

Hence, evaluate
$$\int \frac{x+7}{x^2-x-6}\, dx.$$
[25 marks]

e) Show that the general solution of the differential equation,
$\dfrac{dy}{dx} = \dfrac{6\, sinx}{y}$ is given by
$\frac{1}{2}y^2 = K - 6\cos x$, where K is a constant. [10 marks]
