



UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2026

Module Number: EC3305

Module Name: Signals and Systems (C-23)

[Three Hours]

[Answer all questions, each question carries 10 marks]

Q1 a) A continuous periodic signal $f(t)$ is given by

$$f(t) = \cos\left(2\pi t + \frac{\pi}{4}\right) + \sin(7\pi t)$$

Determine whether the function $f(t)$ is periodic, and if so, find its fundamental period.

[2 Marks]

b) Let $f(t)$ be odd and $v(t) = f(t - 3)$ be causal. Determine all t for which $f(t) = 0$.

[2 Marks]

c) A continuous-time Linear Time-Invariant (LTI) system has the impulse response $h(t)$ given by $h(t) = u(t) - u(t - 1)$. Determine the output $y(t)$ of the system when the input $f(t)$ is given, as shown in Fig. Q1. Use the graphical convolution method to obtain the answer.

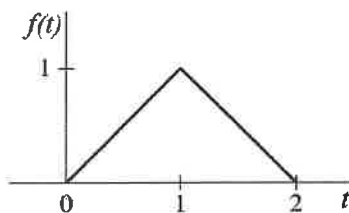


Fig.Q1.

[6 Marks]

Q2 a) Fourier transform of a continuous time signal $f(t)$ is given by $F(\omega)$.

i) State a mathematical expression for $F(\omega)$.

[1 Mark]

ii) Show that the Fourier transform of the signal $f(t)e^{j\omega_0 t}$ is

$$\mathcal{F}[f(t)e^{j\omega_0 t}] = F(\omega - \omega_0)$$

[1 Mark]

iii) Use the result obtained in part a) ii) to find the Fourier transform of the signal

$$h(t) = \frac{2\omega_b}{\pi} \text{sinc}(\omega_b t) \cos(\omega_a t).$$

[3 Marks]

- b) Consider an LTI system which has the impulse response $h(t)$ given in part a) iii) with $\omega_b = 1$ and $\omega_a = 4$. Suppose the input signal for the system is given by

$$f(t) = -1 + 2 \cos(2t) + \cos(4t) - \cos(6t).$$

- Sketch the frequency spectrum of the input signal $f(t)$ and the impulse response $h(t)$ of the system. [2 Marks]
- Use the results obtained in part b) i) to obtain the spectrum of the system response $y(t)$. [2 Marks]
- Find the system response $y(t)$. [1 Mark]

[Hint: Use the Fourier transform pairs in Table 1 to perform the Fourier transform and inverse Fourier transform.]

- Q3 a) i) State a mathematical expression for the Laplace transform of a continuous time signal given by $f(t)$. [2 Marks]
- ii) Derive the Laplace transform of $f(t) = u(t)$ Note that the mathematical derivations are required. [2 Marks]
- b) Consider the resistor-inductor-capacitor (RLC) circuit shown in the Fig. Q3 with input $v_0(t)$ and output $v_1(t)$. Assume all initial conditions are zero.

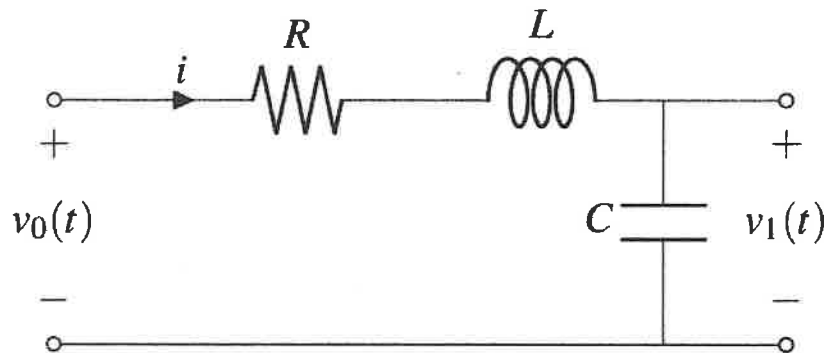


Fig.Q3.

- Convert the time domain circuit shown in Fig.Q3 into its s-domain equivalent using Laplace Transform. [2 Marks]
- Determine the expression for current in s-domain. Assume $R = 3 \Omega$, $L = 1 H$, $C = \frac{1}{2} F$ and $v_0(t) = 10u(t)$. [2 Marks]
- Find the time domain current $i(t)$ for $t \geq 0$ using Inverse Laplace Transform. [2 Marks]

[Hint: Use the unilateral Laplace transform pairs in Table 2 to perform the Laplace transform and inverse Laplace transform.]

- Q4 Consider a continuous time real signal $x(t) = 2\cos(1200\pi t) + \cos(100\pi t)$.
- Find the Nyquist sampling rate (in Hz) and Nyquist interval (in seconds) of the signal.
[2 Marks]
 - If the signal $x(t)$ was sampled through a sampler by multiplying the signal with an impulse train of period $T_s = 0.002 \text{ sec}$. Determine the folding frequency of the sampler and whether the sampler satisfies the Nyquist criteria of the above signal $x(t)$ for a perfect reconstruction.
[1 Mark]
 - Plot the frequency magnitude spectrum $X(\omega)$ of the signal $x(t)$ and the frequency magnitude spectrum $\bar{X}(\omega)$ of the sampled signal $x[n]$ in two separate plots (two rows with x-axis lining up) for frequency range $[-1000\text{Hz}, 1000\text{Hz}]$. Clearly label the impulses.
[4 Marks]
 - If the sampled signal $x[n]$ is reconstructed by passing through an ideal low-pass filter (LPF) with a cutoff frequency of $f_c = 200\text{Hz}$ and a gain of $G = T_s$, Write an expression for the output $y(t)$ of the filter.
[2 Marks]
 - Determine whether the input signal $x(t)$ and the reconstructed signals $y(t)$ are the same. Explain your answer.
[1 Mark]

- Q5 a) The Discrete-Time Fourier Transform (DTFT) can be expressed by its

$$\text{Synthesis Equation: } x[n] = \frac{1}{2\pi} \int_{-\alpha}^{\alpha} X(\omega) e^{j\omega n} d\omega$$

$$\text{Analysis Equation: } X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n] e^{-j\omega n}.$$

- Consider a causal discrete-time signal $x[n]$ given by

$$x[n] = (0.6)^n u[n]$$

Use the DTFT definition to show

$$X(e^{j\omega}) = \frac{1}{1 - 0.6e^{-j\omega}}$$

[2 Marks]

(Hint: Use the geometric series sum formula $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ for $|r| < 1$.)

- Calculate the magnitude $|X(e^{j\omega})|$ at the frequency $\omega = \{0, \pi/2, \pi\}$. Based on the values, determine if this is a low-pass or a high-pass filter.

[2 Marks]

- A causal LTI system is described by the system function

$$H(z) = \frac{z}{z - 0.5}$$

- Determine the poles and zeros of the given function. Plot them on a pole-zero diagram in the z-domain and indicate the Region of Convergence (ROC) on the same plot.

[2 Marks]

- ii) Find the inverse z-transform of the above system. [2 Marks]
- c) If filter response is given by $h[n] = 3\delta[n] + \delta[n - 2]$, determine the output $y[n]$ of the system using z-domain analysis to the input $x[n] = -1\delta[n] + 2\delta[n] - 1\delta[n - 2]$. [2 Marks]

Table 1: Common Fourier transforms

Transform pairs for the CT Fourier transform

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\text{sgn } t$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\text{rect}\left(\frac{t}{T}\right)$	$ T \text{sinc}\left(\frac{T\omega}{2}\right)$
9	$\frac{ B }{\pi} \text{sinc}(Bt)$	$\text{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t), \text{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
11	$t^{n-1}e^{-at}u(t), \text{Re}\{a\} > 0$	$\frac{(n-1)!}{(a + j\omega)^n}$
12	$\text{tri}\left(\frac{t}{T}\right)$	$\frac{ T }{2} \text{sinc}^2\left(\frac{T\omega}{4}\right)$

Table 2: Common unilateral Laplace transforms
Select (Unilateral) Laplace Transform Pairs

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t} u(t)$	$\frac{1}{s - \lambda}$
6	$te^{\lambda t} u(t)$	$\frac{1}{(s - \lambda)^2}$
7	$t^n e^{\lambda t} u(t)$	$\frac{n!}{(s - \lambda)^{n+1}}$
8a	$\cos bt u(t)$	$\frac{s}{s^2 + b^2}$
8b	$\sin bt u(t)$	$\frac{b}{s^2 + b^2}$
9a	$e^{-at} \cos bt u(t)$	$\frac{s + a}{(s + a)^2 + b^2}$
9b	$e^{-at} \sin bt u(t)$	$\frac{b}{(s + a)^2 + b^2}$
10a	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{(r \cos \theta)s + (ar \cos \theta - br \sin \theta)}{s^2 + 2as + (a^2 + b^2)}$
10b	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{0.5re^{j\theta}}{s + a - jb} + \frac{0.5re^{-j\theta}}{s + a + jb}$
10c	$re^{-at} \cos(bt + \theta) u(t)$	$\frac{As + B}{s^2 + 2as + c}$
	$r = \sqrt{\frac{A^2c + B^2 - 2ABa}{c - a^2}}$ $\theta = \tan^{-1} \left(\frac{Aa - B}{A\sqrt{c - a^2}} \right)$ $b = \sqrt{c - a^2}$	
10d	$e^{-at} \left[A \cos bt + \frac{B - Aa}{b} \sin bt \right] u(t)$ $b = \sqrt{c - a^2}$	$\frac{As + B}{s^2 + 2as + c}$

Table 3: Common unilateral z transforms

Select (Unilateral) z-Transform Pairs

No.	$x[n]$	$X[z]$
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\frac{z}{z-1}$
3	$nu[n]$	$\frac{z}{(z-1)^2}$
4	$n^2u[n]$	$\frac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$
7	$\gamma^{n-1} u[n-1]$	$\frac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$n^2\gamma^n u[n]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{n(n-1)(n-2)\cdots(n-m+1)}{\gamma^m m!} \gamma^n u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$ \gamma ^n \cos \beta n u[n]$	$\frac{z(z- \gamma \cos \beta)}{z^2-(2 \gamma \cos \beta)z+ \gamma ^2}$
11b	$ \gamma ^n \sin \beta n u[n]$	$\frac{z \gamma \sin \beta}{z^2-(2 \gamma \cos \beta)z+ \gamma ^2}$
12a	$r \gamma ^n \cos(\beta n + \theta)u[n]$	$\frac{r z[z \cos \theta - \gamma \cos(\beta - \theta)]}{z^2-(2 \gamma \cos \beta)z+ \gamma ^2}$
12b	$r \gamma ^n \cos(\beta n + \theta)u[n] \quad \gamma = \gamma e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma} + \frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r \gamma ^n \cos(\beta n + \theta)u[n]$	$\frac{z(Az+B)}{z^2+2az+ \gamma ^2}$
	$r = \sqrt{\frac{A^2 \gamma ^2 + B^2 - 2AaB}{ \gamma ^2 - a^2}}$	
	$\beta = \cos^{-1} \frac{-a}{ \gamma }$	
	$\theta = \tan^{-1} \frac{Aa-B}{A\sqrt{ \gamma ^2 - a^2}}$	