UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: March 2026

Module Number: EE3306 **Module Name: Signals and Systems (C-23)**

[Three Hours]

[Answer **all questions**, each question carries 10 marks]

Q1 a) A continuous periodic signal $f(t)$ is given by

$$f(t) = cos\left(2\pi t + \frac{\pi}{4}\right) + sin(7\pi t)$$

Determine whether the function $f(t)$ is periodic, and if so, find its fundamental period.

[2 Marks]

b) Let $f(t)$ be odd and $v(t) = f(t-3)$ be causal. Determine all t for which $f(t) = 0$.

[2 Marks]

c) A continuous-time Linear Time-Invariant (LTI) system has the impulse response $h(t)$ given by $h(t) = u(t) - u(t-1)$. Determine the output $y(t)$ of the system when the input $f(t)$ is given, as shown in Fig. Q1. Use the graphical convolution method to obtain the answer.

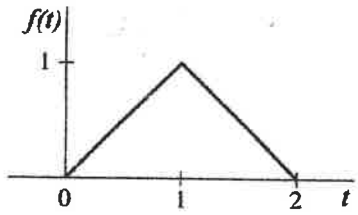

Fig.Q1.

[6 Marks]

Q2 a) Fourier transform of a continuous time signal $f(t)$ is given by $F(\omega)$.

i) State a mathematical expression for $F(\omega)$.

[1 Mark]

ii) Show that the Fourier transform of the signal $f(t)e^{j\omega_0 t}$ is

$$\mathcal{F}\left[f(t)e^{j\omega_0 t}\right] = F(\omega - \omega_0)$$

[1 Mark]

iii) Use the result obtained in part a) ii) to find the Fourier transform of the signal

$$h(t) = \frac{2\omega_b}{\pi} sinc(\omega_b t) cos(\omega_a t).$$

[3 Marks]

b) Consider an LTI system which has the impulse response $h(t)$ given in part a) iii) with $\omega_b = 1$ and $\omega_a = 4$. Suppose the input signal for the system is given by

$$f(t) = -1 + 2\cos(2t) + \cos(4t) - \cos(6t).$$

i) Sketch the frequency spectrum of the input signal $f(t)$ and the impulse response $h(t)$ of the system.

[2 Marks]

ii) Use the results obtained in part b) i) to obtain the spectrum of the system response $y(t)$.

[2 Marks]

iii) Find the system response $y(t)$.

[1 Mark]

[Hint: Use the Fourier transform pairs in Table 1 to perform the Fourier transform and inverse Fourier transform.]

Q3 a) i) State a mathematical expression for the Laplace transform of a continuous time signal given by $f(t)$.

[2 Marks]

ii) Derive the Laplace transform of $f(t) = u(t)$ Note that the mathematical derivations are required.

[2 Marks]

b) Consider the resistor-inductor-capacitor (RLC) circuit shown in the Fig. Q3 with input $v_0(t)$ and output $v_1(t)$. Assume all initial conditions are zero.

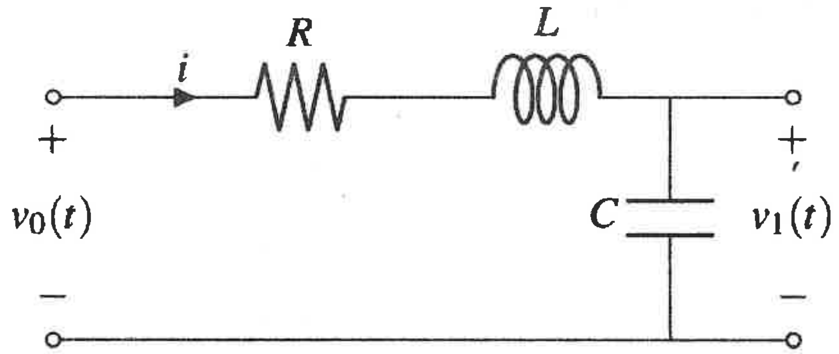

Fig.Q3.

i) Convert the time domain circuit shown in Fig.Q3 into its s-domain equivalent using Laplace Transform.

[2 Marks]

ii) Determine the expression for current in s-domain. Assume $R = 3\ \Omega$, $L = 1\ H$, $C = \frac{1}{2}\ F$ and $v_0(t) = 10u(t)$.

[2 Marks]

iii) Find the time domain current $i(t)$ for $t \geq 0$ using Inverse Laplace Transform.

[2 Marks]

[Hint: Use the unilateral Laplace transform pairs in Table 2 to perform the Laplace transform and inverse Laplace transform.]

Q4 Consider a continuous time real signal $x(t) = 2cos(1200\pi t) + cos(100\pi t)$.

a) Find the Nyquist sampling rate (in Hz) and Nyquist interval (in *seconds*) of the signal.

[2 Marks]

b) If the signal $x(t)$ was sampled through a sampler by multiplying the signal with an impulse train of period $T_s = 0.002\ sec$. Determine the folding frequency of the sampler and whether the sampler satisfies the Nyquist criteria of the above signal $x(t)$ for a perfect reconstruction.

[1 Mark]

c) Plot the frequency magnitude spectrum $X(\omega)$ of the signal $x(t)$ and the frequency magnitude spectrum $\bar{X}(\omega)$ of the sampled signal $x[n]$ in two separate plots (two rows with x-axis lining up) for frequency range $[-1000Hz, 1000Hz]$. Clearly label the impulses.

[4 Marks]

d) If the sampled signal $x[n]$ is reconstructed by passing through an ideal low-pass filter (LPF) with a cutoff frequency of $f_c = 200Hz$ and a gain of $G = T_s$, Write an expression for the output $y(t)$ of the filter.

[2 Marks]

e) Determine whether the input signal $x(t)$ and the reconstructed signals $y(t)$ are the same. Explain your answer.

[1 Mark]

Q5 a) The Discrete-Time Fourier Transform (DTFT) can be expressed by its

Synthesis Equation: $x[n] = \frac{1}{2\pi}\int_{-\alpha}^{\alpha} X(\omega)e^{j\omega n}d\omega$

Analysis Equation: $X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$.

i) Consider a causal discrete-time signal $x[n]$ given by

$$x[n] = (0.6)^n u[n]$$

Use the DTFT definition to show

$$X(e^{j\omega}) = \frac{1}{1 - 0.6e^{-j\omega}}$$

[2 Marks]

(Hint: Use the geometric series sum formula $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ $for\ |r| < 1.$)

ii) Calculate the magnitude $|X(e^{j\omega})|$ at the frequency $\omega = \{0, \pi/2, \pi\}$. Based on the values, determine if this is a low-pass or a high-pass filter.

[2 Marks]

b) A causal LTI system is described by the system function

$$H(z) = \frac{z}{z - 0.5}$$

i) Determine the poles and zeros of the given function. Plot them on a pole-zero diagram in the z-domain and indicate the Region of Convergence (ROC) on the same plot.

[2 Marks]

ii) Find the inverse z-transform of the above system.

[2 Marks]

c) If filter response is given by $h[n] = 3\delta[n] + \delta[n-2]$, determine the output $y[n]$ of the system using z-domain analysis to the input $x[n] = -1\delta[n] + 2\delta[n] - 1\delta[n-2]$.

[2 Marks]

Table 1: Common Fourier transforms

Transform pairs for the CT Fourier transform

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\operatorname{sgn} t$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\operatorname{rect}\left(\frac{t}{T}\right)$	$\|T\| \operatorname{sinc}\left(\frac{T\omega}{2}\right)$
9	$\frac{\|B\|}{\pi}\operatorname{sinc}(Bt)$	$\operatorname{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{1}{a + j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\operatorname{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$
12	$\operatorname{tri}\left(\frac{t}{T}\right)$	$\frac{\|T\|}{2}\operatorname{sinc}^2\left(\frac{T\omega}{4}\right)$

Table 2: Common unilateral Laplace transforms

Select (Unilateral) Laplace Transform Pairs

No.	$x(t)$	$X(s)$
1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$tu(t)$	$\frac{1}{s^2}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$e^{\lambda t}u(t)$	$\frac{1}{s-\lambda}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$
7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$
10a	$re^{-at}\cos(bt+\theta)\,u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$
10b	$re^{-at}\cos(bt+\theta)\,u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$
10c	$re^{-at}\cos(bt+\theta)\,u(t)$ $r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$ $\theta=\tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right)$ $b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$
10d	$e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$ $b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$

Table 3: Common unilateral z transforms

Select (Unilateral) z-Transform Pairs

No.	$x[n]$	$X[z]$
1	$\delta[n-k]$	z^{-k}
2	$u[n]$	$\frac{z}{z-1}$
3	$nu[n]$	$\frac{z}{(z-1)^2}$
4	$n^2u[n]$	$\frac{z(z+1)}{(z-1)^3}$
5	$n^3u[n]$	$\frac{z(z^2+4z+1)}{(z-1)^4}$
6	$\gamma^n u[n]$	$\frac{z}{z-\gamma}$
7	$\gamma^{n-1}u[n-1]$	$\frac{1}{z-\gamma}$
8	$n\gamma^n u[n]$	$\frac{\gamma z}{(z-\gamma)^2}$
9	$n^2\gamma^n u[n]$	$\frac{\gamma z(z+\gamma)}{(z-\gamma)^3}$
10	$\frac{n(n-1)(n-2)\cdots(n-m+1)}{\gamma^m m!}\gamma^n u[n]$	$\frac{z}{(z-\gamma)^{m+1}}$
11a	$\lvert\gamma\rvert^n \cos\beta n\, u[n]$	$\frac{z(z-\lvert\gamma\rvert\cos\beta)}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
11b	$\lvert\gamma\rvert^n \sin\beta n\, u[n]$	$\frac{z\lvert\gamma\rvert\sin\beta}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12a	$r\lvert\gamma\rvert^n\cos(\beta n+\theta)u[n]$	$\frac{rz[z\cos\theta-\lvert\gamma\rvert\cos(\beta-\theta)]}{z^2-(2\lvert\gamma\rvert\cos\beta)z+\lvert\gamma\rvert^2}$
12b	$r\lvert\gamma\rvert^n\cos(\beta n+\theta)u[n] \quad \gamma=\lvert\gamma\rvert e^{j\beta}$	$\frac{(0.5re^{j\theta})z}{z-\gamma}+\frac{(0.5re^{-j\theta})z}{z-\gamma^*}$
12c	$r\lvert\gamma\rvert^n\cos(\beta n+\theta)u[n]$ $r=\sqrt{\frac{A^2\lvert\gamma\rvert^2+B^2-2AaB}{\lvert\gamma\rvert^2-a^2}}$ $\beta=\cos^{-1}\frac{-a}{\lvert\gamma\rvert}$ $\theta=\tan^{-1}\frac{Aa-B}{A\sqrt{\lvert\gamma\rvert^2-a^2}}$	$\frac{z(Az+B)}{z^2+2az+\lvert\gamma\rvert^2}$