# UNIVERSITY OF RUHUNA

**Faculty of Engineering**

End-of-Semester 3 Examination in Engineering: March 2026

**Module Number: CE3205** **Module Name: Structural Analysis I (C-23)**

**[Three Hours]**

[Answer **all questions**; each question carries ten marks]

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Q1 a) Fig. Q1(a) shows a beam with fixed ends subjected to a uniformly distributed load W kN/m. Using the moment curvature relationship, prove that the fixed end moments at A and B, $M_{FAB}$ and $M_{FBA}$ is equal to $Wl^2/_{12}$ *where* $l$ is the length of the beam between support A and B.

[4 Marks]

b) Floor level idealization of a beam in a three-dimensional concrete frame, along with its loading, is shown in Fig. Q1(b). Draw the bending moment diagrams of the beam using moment distribution method. Assume that the beam section is uniform throughout the length, and it has a constant EI for the entire length.

[6 Marks]

Q2. a) Express principles of virtual work on your own wards.

[2 Marks]

b) The propped cantilever beam shown in Fig. Q2 is loaded with a uniformly distributed load W kN/m and a moment couple of W'L kNm (clock wise moment) applied at the center of the beam AB. Assuming a suitable virtual force system and writing an expression for real deformation, calculate all the support reactions using principal of virtual work. Consider stiffness of the beam equal to EI.
(Hint: Consider only the bending work of the system for the calculation)

[4 Marks]

c) Assuming a suitable virtual force system (virtual force system with unit moment applied at the center of the beam AB), calculate the rotation of the center of the beam AB.

[4 Marks]

Q3 a) Express Castigliano's theorem of strain energy and complementary strain energy in your own words.

[2 Marks]

b) Considering the ability of the use of the above theory to generate additional equation to calculate the unknown reaction of the 2D frame structure like the one shown in the Fig. Q3, calculate the all the reaction of the 2D frame structure shown in Fig Q3. Consider only the bending energy. Take Young's modulus of the material E, span of the three interconnected elements equal to $l$ , second moment of area of the section $I$. Also consider that the frame operate liner elastic manner.

[5 Marks]

b) Calculate the horizontal sway at the tip of the vertical element BD at D.

[3 Marks]

Q4 a) Portal frame structure ABCD with uniformly distributed load of 48kN/m applied on beam element BC and a horizontal load of 50kN applied at B is shown in Fig Q4. Using the slope deflection method, calculate horizontal sway and rotation at joints B and C. Take the EI of the column members as $420\times 10^3$ kNm$^2$ and the EI of the beam element as $840\times 10^3$ kNm$^2$.

[6 Marks]

b) Using rotations and sway at B and C, calculate joint moments at A, B, C and D, and hence complete the bending moment diagram of the portal frame ABCD.

[4 Marks]

Q5. a) State two theories involved in deriving the Three Moment Theorem. Using sketches to clarify your explanations, derive the Three Moment Theorem (shown below) starting from the first principles.

$$\frac{M_1 l_1}{E_1 I_1} + 2M_2\left(\frac{l_1}{E_1 I_1} + \frac{l_2}{E_2 I_2}\right) + \frac{M_3 l_3}{E_3 I_3} = 6\left[\left(\frac{\delta_1 - \delta_2}{l_1} + \frac{\delta_3 - \delta_2}{l_2}\right) - \left(\frac{A_1 a_1}{E_1 I_1 l_1} + \frac{A_2 a_2}{E_2 I_2 l_2}\right)\right]$$

Note: All notation have their usual meanings

[4 Marks]

b) Fig. Q5 shows a straight elastic beam ABC of uniform cross section with constant flexural rigidity of EI (=820 $10^5$kNm2) subjected to a uniformly distributed load of 30kN/m. Beam connected to soil encounters uniform subgrade reaction of 20,000 kN/m at points A, B and C (Consider this as spring constant at ABC). Considering the reaction at ABC as $R_a$, $R_b$ and $R_c$ (unknowns), calculate the joint moment B using the three-moment theory and complete the bending moment diagram of the beam ABC.

[6 Marks]

Q6. a) Explain the differences between the action diagram and the influence line of any function (i.e. bending moment, shear force, reaction). Also, explain the advantages of the influence line over the action diagram.

[2 Marks]

b) Fig.Q6 b shows a continuous beam ABC with a hinge at point D, a quarter of the beam BC. Derive expressions for support reactions at A, B and C when the unit load move across the beam ABC and draw the influence line for the reactions at point A.B and C.

[4 Marks]

c) Point E is located 6 m right of the support A (Middle of beam AB) as shown in Fig. Q6 c. Based on the support reactions plot, the influence line of the bending moment and shear force at E.

[2 Marks]

d) Location of two axle loads 4 kN each passing on the beam ABC is given in Fig. Q6d. Based on the location given in Fig. Q6d, calculate reactions at joint A, B and C and bending moment and shear force at E.

[2 Marks]

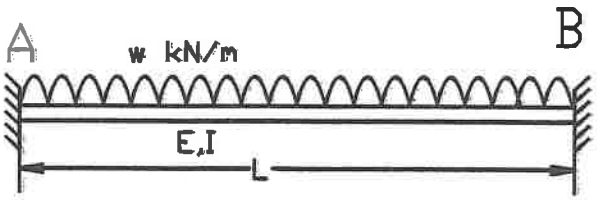


Fig Q1a

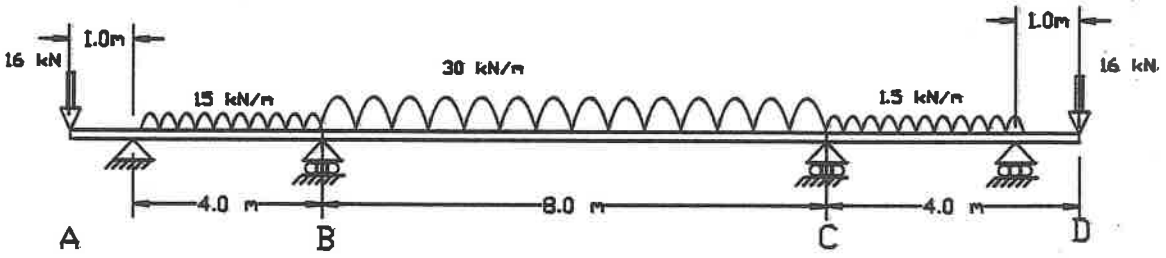


Fig. Q1b

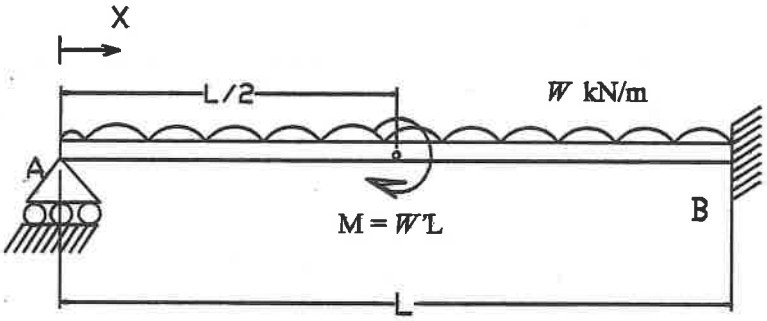


Fig. Q2

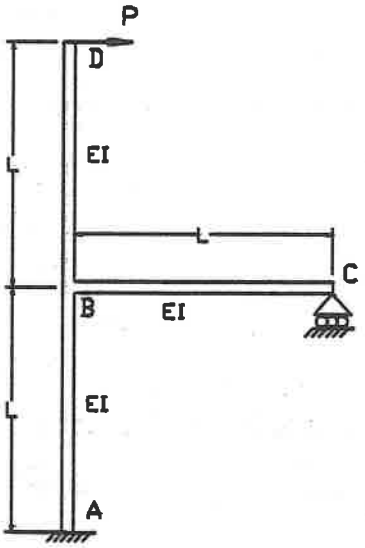


Fig Q3

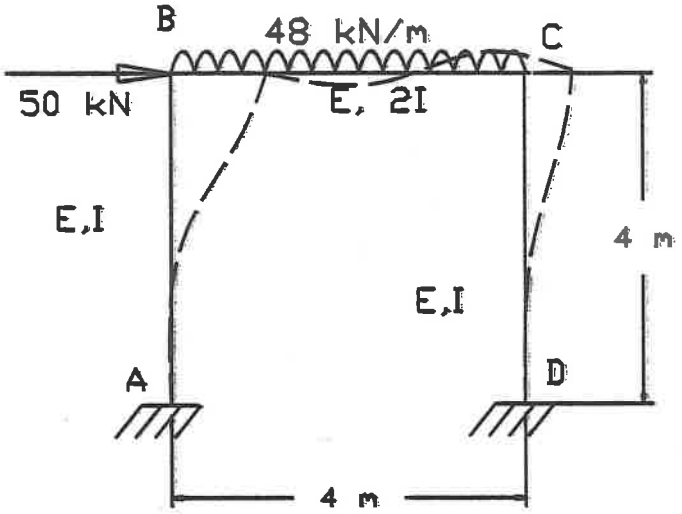


Fig. Q4

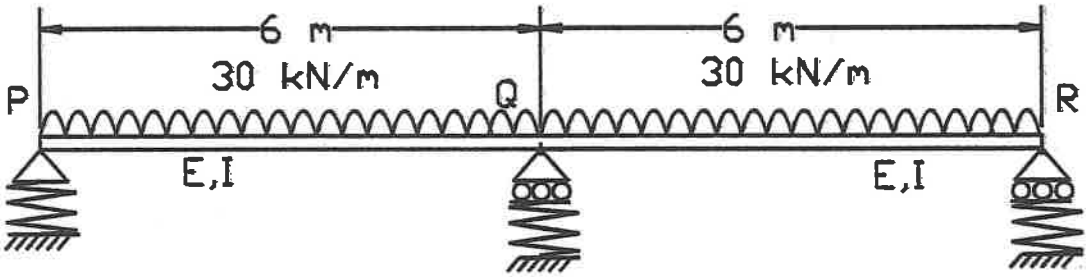


Fig. Q5

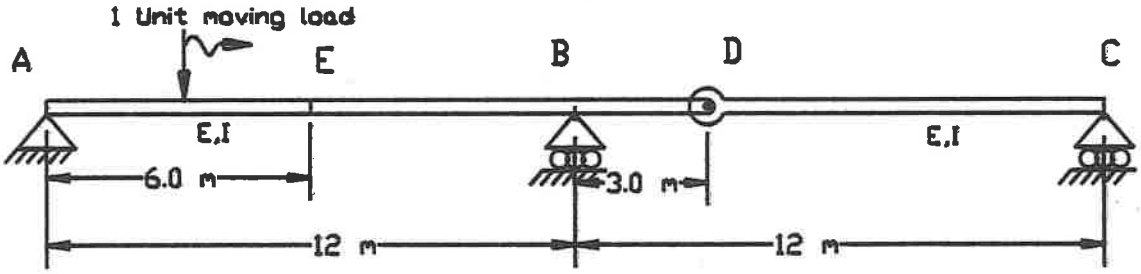


Fig Q6c

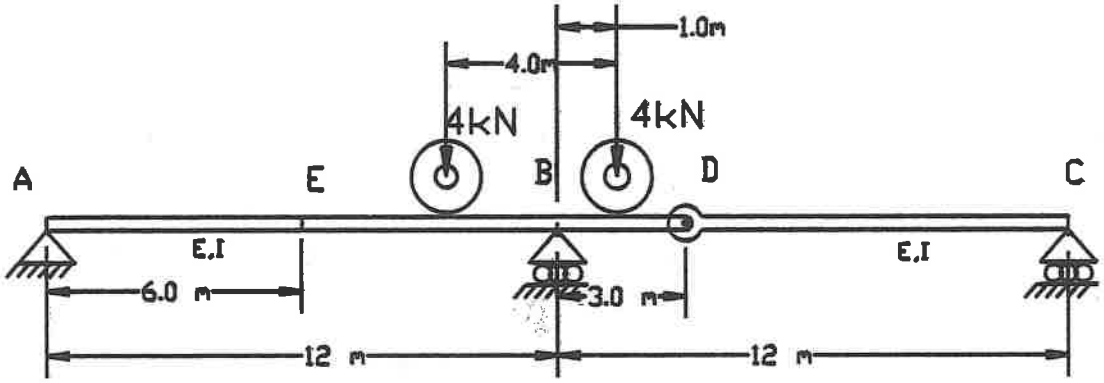


Fig Q6d