UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 2 Examination in Engineering: March/ April 2026

Module Number: IS2401 **Module Name: Linear Algebra and Differential Equations (C – 23)**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1 a) i. Verify that the following differential equation is homogeneous. Then, find its general solution.

$$x^2dy - y(x-y)dx = 0$$

ii. Solve the following linear differential equation

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{\tan^{-1} x}{1+x^2} \ ; \ y(0) = 1$$

[4 Marks]

b) i. State Newton's law of cooling in the form of a differential equation.

ii. A dead body is found at $12{:}\,00\ p.m.$ in a room that is maintained at 72^oF. If the dead body is 82^oF when it is found and has cooled to 80^0F at $1{:}\,00\ p.m.$, estimate the time of death.

Assume a living body maintains a temperature of 98.6^oF.

(Use, $\ln 0.8 = -0.22$ and $\ln 2.66 = 0.97$)

[8 Marks]

Q2 a) Find a Particular Integral of the following differential equation.

$$(D^3 + D^2 - D - 1)y = -5\cos 2x$$

Hence find the general solution of

$$\frac{d^3 y}{dx^3} + \frac{d^2 y}{dx^2} - \frac{dy}{dx} - y = 25\cos 2x$$

[5 Marks]

[Q2 Cont. in Page - 2]

b) Define,

i. An ordinary point

ii. A regular singular point,

of a differential equation of the form $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$

[2 Marks]

c) Check whether singular point of the following differential equation is regular or irregular.

Then find the solution accordingly.

$$2\,x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1 - x)y = 0$$

[5 Marks]

Q3 a) Evaluate $\int_C \mathbf{F}.\mathbf{dr}$ along the following path C, where

$$\mathbf{F} = (2xy + z^3)\,\mathbf{i} + x^2\,\mathbf{j} + 3xz^2\,\mathbf{k}\,.$$

i. The polygonal path consisting of the line segments from (0,0,0) to (0,0,1), from (0,0,1) to (0,2,1) and from (0,2,1) to (2,2,1).

ii. Once around the circle $x^2 + y^2 = 4$, in the xy plane.

[4 Marks]

b) i. Define a conservative vector field

ii. For a scalar function ϕ, if $\mathbf{F} = \nabla\phi$, show that $\mathbf{F}$ is a conservative field. Hence, deduce that the work done in moving a particle is independent from the path.

iii Show that the vector field $\mathbf{F} = y^2z^3\,\mathbf{i} + 2xyz^3\mathbf{j} + 3xy^2z^2\mathbf{k}$ is a conservative and hence, find the scalar potential function ϕ, such that $\mathbf{F} = \nabla\phi$

[5 Marks]

c) Evaluate $\iint_S \mathbf{F}.\mathbf{n}\,dS$ where $\mathbf{F} = yz\,\mathbf{i} + zx\,\mathbf{j} + xy\,\mathbf{k}$ S is that part of the surface of sphere $x^2 + y^2 + z^2 = 1$ lies on the first octant and $\mathbf{n}$ is the unit outward drawn normal to the surface S.

[3 Marks]

Q4 a) i. State the Gauss' Divergence theorem.

ii. Use Gauss' Divergence theorem to evaluate the surface integral $\iint_S \mathbf{F} . dS$ where, $\mathbf{F}$ is the vector field $x^2y\,\mathbf{i} + 2xy\,\mathbf{j} + z^3\mathbf{k}$ and S is the surface of the unit cube $0 \leq x \leq 1, 0 \leq y \leq 1$ and $0 \leq z \leq 1$.

iii. Show that $\iint_S \mathbf{F}.\mathbf{n}\, dS = \frac{12\pi a^5}{5}$, where $\mathbf{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and S is the sphere $x^2 + y^2 + z^2 = a^2$ and $\mathbf{n}$ is the unit outward drawn normal to the surface S.

[6 Marks]

b) i. State the Stokes' theorem.

ii. Verify stokes' theorem for a vector field defined by $\mathbf{F} = (x^2 - y^2)\mathbf{i} + 2xy\,\mathbf{j}$ in a rectangular region in the xy plane bounded by the lines $x = 0, x = a,$ $y = 0$ and $y = b$

[6 Marks]

Q5 a) State whether each of the following is 'True' of 'False'. Justify your answers

i. $V = \{(x, y) \in \mathbb{R}^2 | \, x^2 + y^2 \leq 1\}$ is a vector space.

ii. W consists of all polynomials with integer coefficients. W is a subspace over vector space of real polynomials.

iii. $S = \{(1,1,0,1), (1,0,1,1), (1,-1,0,2), (2,1,1,0)\}$ is a basis of $\mathbb{R}^4$.

[3 Marks]

b) i. If V and U are two vector spaces, define Kernel, Image, Nullity and Rank of a linear transformation $T: V \to U$.

ii Find basis and dimension of $Ker(T)$ and $Im(T)$ linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^4$ is defined as follows,

$T\,(x, y, z) = (x + y + z,\ y + z,\ x + z,\ x + y)$

[4 Marks]

c) Find the eigenvalues and the corresponding eigenvectors of

$$A = \begin{bmatrix} -4 & -6 & -7 \\ 3 & 5 & 3 \\ 0 & 0 & 3 \end{bmatrix}.$$

Hence, diagonalize the matrix A.

Find the eigenvalues of A^3 and A^{-1}.

[5 Marks]

..End..