UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 2 Examination in Engineering: March/ April 2026

Module Number: IS2401 **Module Name: Linear Algebra and Differential Equations (C – 18)**

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1 a)

i. Show that the following is an exact differential equation, and hence, find its general solution

$$(2x + \sin y)\,dx + (x\cos y - 2y)\,dy = 0$$

ii. Solve the following Initial Value Problem.

$$\frac{dy}{dx} + \frac{2}{x}y = x\ln x\,; \quad y(1) = 2$$

[4 Marks]

b)

i. Find the general solution of the second-order linear differential equation:

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}; \quad y(0) = -\frac{1}{2}, \quad \left.\frac{dy}{dx}\right|_{x=0} = \frac{3}{2}$$

ii. Use Frobenius method to obtain two linearly independent solutions about $x = 0$ of the following differential equation

$$2x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x^2 + 1)y = 0.$$

[8 Marks]

Q2 a)

i. Define a conservative vector field.

ii. For a scalar function ϕ, if $\mathbf{A} = \nabla\phi$, show that $\mathbf{A}$ is a conservative vector field.

[2 Marks]

b) If $\mathbf{A} = (x + 2y)\,\mathbf{i} - 2xz\,\mathbf{j} + yz^2\,\mathbf{k}$, find the work done in moving a particle;

i. along the path C given by $x = t, y = 2t$ and $z = t^2$, where $0 \le t \le 1$.

ii. once around a circle C in the xy plane with its center at the origin and radius of 2.

[4 Marks]

[Q2 Conts. on Page-2]

c) Let S be the curved surface of the cylinder $x^2 + y^2 = 4$, with $0 \le z \le 5$, open from both top and bottom. Find the surface integral of the vector field $\mathbf{F} = z\mathbf{i} - x\,\mathbf{j} + 2yz\ \mathbf{k}$ outward through the surface S.

[3 Marks]

d) Let $A = xy^2\mathbf{i} - 2x(y-z)\,\mathbf{j} + 2zy\,\mathbf{k}$. Compute the volume integral

$$\int_V \nabla.\mathbf{A}dV,$$

where V is the volume of the cuboid bounded by the planes $x = 0, x = 1$, $y = 0,\ y = 2,\ z = 0$, and $z = 1$.

[3 Marks]

Q3 a) i. State the divergence theorem

ii. Let $\mathbf{A} = 3x\mathbf{i} - 2xz^2\,\mathbf{j} + z\,\mathbf{k}$. Use divergence theorem to evaluate

$$\int_S \mathbf{A}.\mathbf{n}dS,$$

where S is the surface of a solid sphere of radius 2 centered at the origin (0,0,0).

[4 Marks]

b) If S is any closed surface enclosing a volume V and $\mathbf{n}$ is the unit outward drawn normal to the surface S, use the divergence theorem to show that,

$$\iint_S \phi\mathbf{n}dS = \iiint_V \nabla\phi dV.$$

Hence, deduce that

$$\iint_S r^4\mathbf{n}dS = \iiint_V 4r^2\mathbf{r}dV$$

[4 Marks]

c) i. State the Stokes' theorem

ii. Use Stokes' theorem to evaluate

$$\iint_S (\nabla \times \mathbf{A}).\mathbf{n}\,dS$$

Where $\mathbf{A} = 2z^2\mathbf{i} + xy^2\,\mathbf{j} - (x^2 + 4y)\,\mathbf{k}$ and S is the open hemisphere defined by $x^2 + y^2 + (z-2)^2 = 4$ lying above the plane $z = 2$.

[4 Marks]

Q4. a) Let V be a vector space over the scalar field F. Define

i. Subspace of V.

ii. Basis and dimension of V.

[2 Marks]

b) State whether each of the following is true or false. Justify your answers.

i. $W = \{(a, b, c): a \geq 0\}$ is a subspace of $\mathbb{R}^3$ under usual addition and scalar multiplication.

ii. If U and W are subspaces of a vector space, V then $U \cap W$ is also a subspace of V.

iii. $S = \{(2, -1, 1,0),\ (1, -1,1,1),\ (1,0,0, -1),\ (3, -2,2,1)\ \}$ is a linearly independent set.

iv. $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that $T(x, y) = (x + y, x - y, x)$ is a linear transformation

[6 Marks]

c) Let U and W be subspaces of $\mathbb{R}^4$ generated by the sets
$S = \{(1, -1,0,2), (2,0,1,1), (1,1,1, -1), (1, -3, -1,3)\}$ and
$T = \{(1,1,1, -1), (0, -2, -1,3), (1, -1,0,2), (2,0,2,1)\}$ respectively.

i. Find bases and the dimensions of U and W

ii. Find a basis and the dimension of $U + W$

iii. Find the dimension of $U \cap W$

[4 Marks]

Q5 a) i. If V and U are two vector spaces, define the 'Kernel' and the 'Image' of a linear transformation $T: V \to U$.

ii. Explain the meaning of the terms the 'Nullity' and the 'Rank'.

iii. Let $T: \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation given by

$$T(x, y, z, t) = (x + y + z,\ \ x - z + t,\ \ 2x + y + t)$$

Then, find the nullity and the rank of T.

[4 Marks]

b) Let λ be an eigenvalue of the matrix A. Show that

i. λ^2 is an eigenvalue of A^2; $\quad n \in \mathbb{N}$

ii. $\dfrac{1}{\lambda}$ is an eigenvalue of A^{-1}, where $\lambda \neq 0$.

[2 Marks]

[Q5 Conts. on Page-4]

c) Find the eigenvalues and the corresponding eigenvectors of

$$A = \begin{bmatrix} 2 & -3 & -3 \\ 6 & -7 & -6 \\ -6 & 6 & 5 \end{bmatrix}.$$

Is the matrix A diagonalizable. Justify your answer.

Find the eigenvalues of A^2 and A^{-1}.

[6 Marks]

...End...