



Modeling Conditional Value-at-Risk Using Tukey's g-and-h Family of Distributions with Maximum Approximated Likelihood Estimation

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The main purpose of this study was to model Conditional Value-at-Risk using Tukey's g-and-h distribution and employ the maximum approximated likelihood method for parameter estimation, subsequently contrasting Tukey's distribution with Cornish-Fisher and classical approaches. While the Cornish-Fisher method modifies the normal quantile to account for skewness and kurtosis, the classical methodology assumes that asset returns follow a normal distribution. Historical daily return data from January 1, 2003, to January 17, 2011, from Spain were used in the study. Conditional Value-at-Risk (CVaR), also known as Expected Shortfall (ES), is a risk indicator that shows the expected loss at a given confidence level when a loss exceeds the Value-at-Risk. The maximum approximated likelihood estimator is one approach used to estimate the parameters of Tukey's distribution. Because of its light-tailed assumption, the classical technique results in the least extreme losses. Due to its sensitivity to skewness and kurtosis modifications, the Cornish-Fisher approach produces noticeably larger tail losses; however, at very high confidence levels, this may introduce instability. Without the dramatic divergence observed in the Cornish-Fisher method, the MALE-based Tukey's model captures heavy-tailed traits with smoother and more stable tail behavior. These findings imply that MALE provides a more reliable paradigm for measuring high risk. These findings have applications in stress testing, regulatory calculation, and risk management, where precise extreme tail risk estimation is essential. Model errors, parameter uncertainty, data selection, and inadequate data may still impact the proposed strategy. Consequently, even while MALE enhances tail risk prediction, its efficacy depends on adequate data and trustworthy estimation techniques. As a recommendation, to further confirm the MALE model's resilience in measuring severe risk, additional research should examine its performance in various market scenarios and on larger datasets.

Keywords: Classical method, Conditional Value-at-Risk, Cornish-Fisher method, Maximum approximated likelihood method